Multifractal Analysis of Brazilian Agricultural Commodities

Ivana Stosic
Department of Economics, Vanderbilt University
Graduate Program in Economic Development
VU Station B #351828, 2301 Vanderbilt Place, Nashville, Tennessee 37235-1828.
E-mail: ivana.stosic1@gmail.com

Alessandro H. S. Santos, Tatijana Stosic
Departamento de Estatística e Informática, UFRPE,
52171-900, Rua Dom Manoel de Medeiros, s/n, Dois Irmãos, PE
E-mail: alessandro.sssantos@ufpe.br, tastosic@gmail.com

Keywords: Multifractal Detrended Fluctuation Analysis, Agricultural Commodities, Long-term correlations

Abstract: We investigate multifractal properties of daily prices of Brazilian agricultural commodities, using Multifractal Detrended Fluctuation Analysis (MF-DFA). We find that return temporal series belong to a class of multifractal processes. The degree of multifractality measured by the range of generalized Hurst exponent varies from one commodity to another, with stronger multifractality for rice, calf and cattle prices than for sugar, coffee, corn and soybean prices. Our results contribute to evaluation of efficiency of the Brazilian commodity market.

1 Introduction

The complexity of financial markets is the result of a large number of interacting units that are affected by various internal and external factors. Although these factors may vary from market to market, some variables generated by underlying stochastic processes, such as the values of market indices, stock returns, share volume, and number of trades, display universal behavior [4]. One of the important features of market dynamics that reflects this universality is the existence of auto-correlations in financial time series and cross-correlations between financial variables [2]. However, while stock and currency price changes have been studied extensively, commodity price data are less accessible and have received significantly less attention in the literature. In this work we apply multifractal analysis to investigate power-law correlations of Brazilian agricultural commodities. We use the Multifractal Detrended Fluctuation Analysis (MF-DFA) method [1], which allows for the quantification of higher order correlations in non-stationary temporal series.

2 Data and Methodology

In this work we use data on daily prices of seven agricultural commodities (sugar, rice, calf, cattle, soybean, corn and coffee), recorded over the period 1996-2012 and obtained from http://cepea.uesp.br. We analyze normalized price returns \( r_t = (\ln P_{t+1} - \ln P_t) / \sigma \) where \( P_t \) is the price, and \( \sigma \) is the standard deviation of \( \ln P_{t+1} - \ln P_t \) over the duration of time series.
Multifractal Detrended Fluctuation Analysis

The MF-DFA procedure is briefly described as follows. The first step is integration of the original series \( x(i), i = 1, \ldots, N \) to produce \( X(k) = \sum_{i=1}^{k} [x(i) - \langle x \rangle] \), \( k = 1, \ldots, N \), where \( \langle x \rangle = \frac{1}{N} \sum_{i=1}^{N} x(i) \) is the average. Next, the integrated series \( X(k) \) is divided into \( N_n \) non-overlapping segments of length \( n \) and in each segment \( s = 1, \ldots, N_n \) the local trend \( X_S(k) \) (linear or higher order polynomial least square fit) is estimated and subtracted from \( X(k) \). The \( q \) th order fluctuation function is calculated as

\[
F_q(n) = \left\{ \frac{1}{N_n} \sum_{s=1}^{N_n} \left[ \frac{1}{n} \sum_{k=(s-1)n+1}^{sn} [X(k) - X_s(k)]^2 \right]^{q/2} \right\}^{1/q}
\]

where, in general, \( q \) can take any real value except zero. If long-term correlations are present, \( F_q(n) \) increases with \( n \) according to a power law \( F_q(n) \sim n^{h(q)} \). The generalized Hurst exponent \( h(q) \) is obtained as the slope of the regression (least square line fitting) of \( \log F_q(n) \) versus \( \log n \). For monofractal time series \( h(q) \) is independent of \( q \), while for multifractal time series, for which small and large fluctuations scale differently, \( h(q) \) is a decreasing function of \( q \). The multifractality degree can be quantify by \( \Delta h = h(q)_{\text{max}} - h(q)_{\text{min}} \) where higher values of \( \Delta h \) correspond to larger variability in scaling of fluctuations of different size [1].

3 Results

We calculate generalized Hurst exponents for each considered agricultural commodity, and find that \( h(q) \) is a decreasing function of \( q \) in all cases, indicating multifractal behavior of price fluctuations (Fig. 1). Table 1 shows the degree of multifractality \( \Delta h \) for the seven analyzed series. The results show that rice, calf, and cattle prices are characterized by a higher degree of multifractality, indicating that price fluctuations of these commodities are more clustered. Higher level of price clustering may be caused by slower price response to demand, which indicates decreased market efficiency [3].

<table>
<thead>
<tr>
<th>Sugar</th>
<th>Rice</th>
<th>Calf</th>
<th>Cattle</th>
<th>Coffee</th>
<th>Corn</th>
<th>Soybean</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.406</td>
<td>0.674</td>
<td>0.664</td>
<td>0.751</td>
<td>0.380</td>
<td>0.481</td>
<td>0.540</td>
</tr>
</tbody>
</table>

Table 1 Multifractality degree \( \Delta h \) for the considered agricultural commodities.
4 Conclusion

In this work we apply Multifractal Detrended Fluctuation Analysis (MF-DFA) to analyze dynamics properties of Brazilian agricultural commodities prices. Our results show that daily return temporal series belong to a class of multifractal process. We also find the higher degree of multifractality for rice, calf and cattle prices, indicating that price fluctuations of these commodities are more clustered, which is characteristic of less efficient markets.

References


