Hybrid Metaheuristics for Crop Rotation

Angelo Aliano Filho*
Doutorando em Matemática Aplicada, IMECC, UNICAMP, 13083-859, Campinas, SP, Brazil

Helenice de Oliveira Florentino†
Departamento de Bioestatística, IB, UNESP, 18618-970, Botucatu, SP, Brazil

Margarida Vaz Pato‡
CIO, Faculdade de Ciências da Universidade de Lisboa and ISEG,
Universidade Técnica de Lisboa, Rua do Quelhas, nº 6, 1200-781, Lisboa, Portugal.

Abstract: This work presents a mathematical model adapted from literature for the Crop Rotation Problem with Demand Constraints (CRP-D). This highly complex combinatorial problem is devised to find a vegetable planting program that takes into account green fertilization restrictions, the set-aside period, planting restrictions for neighboring lots and for crop sequencing, demand constraints, while, at the same time, maximizes the profitability of the planting area. The main aim of this study was to test hybrid metaheuristics and their performance in a real context. Two hybrid algorithms were developed in this work: a Genetic Algorithm with Simulated Annealing and a Memetic algorithm. A new constructive heuristic was also developed to provide initial solutions for the metaheuristics. A computational experiment was performed with a medium dimension real instance. The computational results showed that these algorithms determined feasible solutions in a short computational time, compared with the time spend to get the optimal solution, thus proving their efficacy in dealing with this practical application of the CRP-D.

Keywords: Optimization, Hybrid Metaheuristics, Crop Rotation.

1 Introduction

Brazil has an agricultural tradition, and currently a variety of grains and vegetables are planted on a large scale. Natural conditions, such as climate, fertile soil and relief, foster the development of this important economic sector.

Crop rotation problems have been studied by many authors, using linear, mixed and nonlinear optimization. For example, the model presented by [1] is a mixed-integer linear programming problem that includes demand constraints. In [2] a nonlinear goal programming model with binary variables was developed. In [3] a binary linear model for crop rotation was developed, taking into account neighbors,
sequence of the same culture, green manure and set aside period constraints, and which objective was to maximize the occupation of the planting area. All these studies solved the problem using different strategies based on exact optimization techniques.

In this work, considering the complexity and the combinatorial nature of the Crop Rotation Problem with Demand Constraints (CRP-D) we proposed two hybrid metaheuristics, both embedding a constructive heuristic.

2 Mathematical Model

The crop rotation problem studied by [3] aimed at maximizing occupation of a specific planting area with the following rotation constraints for the planning horizon: (a) sowing season, (b) continuity for same-family crops, (c) neighboring for same family crops, (d) green fertilization, (e) set-aside period. Here we studied CRP-D, a crop rotation problem with the objective of maximizing profitability and with a new type of restraint: (f) demand constraint.

Let us consider the planning horizon divided into \( M \) periods of similar duration, a set of \( N \) crops belonging to \( N_f \) plant families and the planting area with \( L \) lots. Other parameters follow:

- \( C \): set of trade crops;
- \( A \): set of crops for green fertilization;
- \( F_p \): set of plants of the family \( p, p = 1, ..., N_f \);
- \( t_i \): planting cycle of crop \( i \), including soil preparation and harvesting (in month);
- \( l_{ij} \): profitability of crop \( i \) in period \( j \) per unit of area (R$/ha);
- \( I_i = [E_i, T_i] \): crop planting interval, for crop \( i \), in which \( E_i \) is the earlier period and \( T_i \) is the later period (in month);
- \( p_{ij} \): production of crop \( i \) in period \( j \) (production unit/ha);
- \( D_i \): demand for crop \( i \) (production unit/ha);
- \( I_i^D = [E_i^D, T_i^D] \): demand interval, for crop \( i \), in which \( E_i^D \) is the earlier period and \( T_i^D \) is the later period (in month);
- \( S_k \): set of lots adjacent to lot \( k \);
- \( area_k \): area of lot \( k \) (ha).

The decision variables were defined as follows:

\[
x_{ijk} = \begin{cases} 
1, & \text{if crop } i \text{ is planted in period } j \text{ in lot } k \\ 
0, & \text{otherwise.}
\end{cases}
\]
A binary linear programming model for CRP-D is described below:

$$\text{maximize } z = \sum_{i \in C} \sum_{j \in I_i} \sum_{k=1}^{L} \text{area}_k \cdot l_{ij} \cdot x_{ijk}$$  \hspace{1cm} (1)$$

subject to

$$\sum_{i \in F_p} \sum_{r=0}^{t_i-1} \sum_{v \in S_k} x_{(j-r)v} \leq L \left( 1 - \sum_{i \in F_p} \sum_{r=0}^{t_i-1} x_{(j-r)k} \right), p = \{1, ..., N_f\}, j = \{1, ..., M\}, k = \{1, ..., L\}$$  \hspace{1cm} (2)$$

$$\sum_{i \in F_p} x_{(j-r)k} \leq 1, \quad p = \{1, ..., N_f\}, j = \{1, ..., M\}, k = \{1, ..., L\}$$  \hspace{1cm} (3)$$

$$\sum_{i=1}^{N+1} \sum_{r=0}^{t_i-1} x_{(j-r)k} \leq 1, \quad j = \{1, ..., M\}, k = \{1, ..., L\}$$  \hspace{1cm} (4)$$

$$\sum_{i \in A} \sum_{j=1}^{M} x_{ijk} \geq 1, \quad k = \{1, ..., L\}$$  \hspace{1cm} (5)$$

$$\sum_{j=1}^{M} \sum_{k=1}^{L} \text{area}_k \cdot p_{ij} \cdot x_{ijk} \geq D_i, \quad i \in C$$  \hspace{1cm} (7)$$

$$x_{ijk} \in \{0, 1\}, \quad i = \{1, ..., N+1\}, \quad j \in I_i, \quad k = \{1, ..., L\},$$  \hspace{1cm} (8)$$

where, for convenience of notation, the set-aside period is represented by crop \( n = N + 1 \) and if \( j - r \leq 0 \) then \( j - r \) is replaced by \( j - r + M \).

The model studied in this work was adapted from the one proposed by [3]. Here, we took into account a new objective function (1), we changed constraints (5) and (6), considering the inequality “≥”, and included demand constraints (7). Constraints (2), (3) and (4) were shared by this model and by the one proposed by [3].

### 3 Methodologies

#### 3.1 The Constructive Heuristic

With the purpose of more efficiently exploring the set of solutions, the initial population of the metaheuristics was determined by a new constructive heuristic. It randomly builds a solution for the CRP-D lot by lot by imposing the planting sequence constraint on the same lot, that is, rotation conditions (a) and (b).

The previous model (1) to (8) uses binary variables. However, it was more convenient to base the algorithms on integer decision variables taking values in the interval \([1, N+1]\). Otherwise, modeling with binary variables would result in a higher dimension codification and enhanced difficulties in dealing with the constraints. Hence, each solution of the problem, represented by \( S \), was associated with an integer \( L \times M \) matrix. Its element \((k, j)\) belongs to \([1, N+1]\) and identifies which crop is being planting on lot \( k \) and in period \( j \), for all \( k = \{1, ..., L\} \) and \( j = \{1, ..., M\} \).
The constructive heuristic provided, for the metaheuristics, initial feasible solutions regarding continuity for same-family crops' constraints (3), since these restrictions were the most difficult to be satisfied. As a result of the application of the constructive heuristic, the metaheuristics proposed only had to deal with constraints (c), (d), (e) and (f).

3.2 The hybrid approaches

Lately, many studies have shown advantages in working with hybrid methods. For example, the authors [4] and [5] adopted this strategy to solve combinatorial problems. Due to the high complexity of this problem and the need to achieve better results, we present two hybrid metaheuristics: a Genetic Algorithm with Simulated Annealing (GA+SA) and a Genetic Algorithm with Local Search, known as Memetic heuristic (M).

The basis of GA+SA was the following: first, we start with GA and after some generations, a solution $\tilde{S}$ is obtained (the best one of the generation). Then $\tilde{S}$ is the initial solutions to the SA, which finishes the search. The metaheuristic M has a similar process: the GA provided a (good) initial solution for the Local Search. This Local Search took into account $\tau L$ elements in the neighborhood, where $\tau$ was a control parameter. In addition, this search stops when the number of movements exceeds $\tau L$.

Therefore, with these hybrid procedures we reduced the “strength” of each heuristic component and we built a new method, in general, more efficient than the pure approaches.

4 Computational experiments

The experiments were performed with the algorithms coded using Matlab software, version 7.4.0 R2007a, on Core 2 Quad microcomputers with 2GB memory and 250 GB hard-disk memory at the Department of Biostatistics of UNESP in Botucatu, Brazil. In this work, each metaheuristic algorithm proposed ran 20 times.

The experiments took into account a real instance of CRP-D with a planting area with 16 non-parallel lots, 29 cultures belonging to 11 different botanical families, in a planning horizon of two years.

To access the performance of these metaheuristics in this context, we measured the average CPU time per run ($t$ in seconds), the time elapsed to get a feasible solution ($t_f$), the number of times the algorithm provided the (feasible) best solution out of 20 ($\mu$) and, this case, we calculate the average profitability found per run ($\bar{z}$ in R$\$,), and the best profitability ($z_{best}$). The value $\lambda = \frac{\bar{z}}{\sigma}$, where $\sigma$ is the standard deviation relative to $\bar{z}$, represents the coefficient of relative variation. At last, the value $\Delta z$ represents the average relative deviation between the profitability of the first best and final best solutions found at each run of the algorithm. These last four figures were determined with the purpose of accessing the improvement achieved with the algorithm from its beginning to the end.

To evaluate the quality of the solutions obtained from the metaheuristics, we solved the problem with real data using the software GLPK 4.47, thus determining the optimum value $z^* = R\$ 4,115,232.00, in a computational time of 7.38 hours, using the same computers.

The results obtained are displayed in Table (1).
We may see that the hybrid algorithms provided good results for this problem, mainly they showed the ability to get feasible solutions in a reasonable computational time. More precisely, the hybrid GA+SA had a performance slightly better than the M. In fact, it used 68% of time the of M and its $\bar{z}$ value was 2.6% higher. Moreover, the figure $z_{best}$ obtained by AG+SA achieved 71% of $z^*$, while for M this proportion was 69%. These figures showed that GA combined with SA was more efficient than GA with Search Local.

In Figure (1), we exhibit a particular simulation of the metaheuristics GA+SA and M with 1,000 iterations each. The horizontal axis denotes the iterations and the vertical axis the profitability.

![Figure 1: Evolution of the best solution from the methods with 1,000 iterations](image)

Here, GA+SA ran with 420 iterations (generations) for GA and 580 iterations for SA. The M ran with 744 iterations for GA and 256 iterations for Search Local. The arrows in the figure indicate when GA ends its search and the other method starts its operation. We notice the advantage of using hybrid methods. When the first approach got stuck on a population of similar solutions, the second approach started the search for a local optimal solution, naturally with higher profit.

## 5 Some Conclusions

In this study, we presented a model for crop rotation based on the one proposed by [3]. We incorporated new demand constraints, because demand is very important from the practical perspective, insofar as these constraints possess perfect real applicability taking into account both technical as well as economic considerations.

As the model has many constraints and variables and it has a combinatorial nature, we decided to solve it using metaheuristics. These approaches, besides providing good solutions, required a low computational expenses.
Two different metaheuristics were proposed: a hybrid GA with SA and a Memetic heuristic. Moreover, we considered a new constructive heuristic procedure, with the intention of building feasible solutions with respect to the consecutive period constraint and thus, to enforcing feasibility during the optimization process. This heuristic revealed a significant effect in the computational results.

With these experiences we showed that these metaheuristics may be viable, simple and efficient approaches for this highly complex combinatorial problem.

Referências


