Robust Flutter Analysis Including Structural Uncertainties

Douglas D. Bueno, Clayton C. Marqui, Luiz C. S. Góes,
Instituto Tecnológico de Aeronáutica - ITA
15054-000, São José dos Campos, SP
E-mail: ddbueno@ita.br, crmarqui@hotmail.com, goes@ita.br,

Paulo J. P. Gonçalves
Faculdade de Engenharia de Bauru, UNESP
Dep. de Engenharia Mecânica
E-mail: paulo.jpg@feb.unesp.br

Abstract: The analysis of aeroelastic flutter is a main topic in the aeronautic industry. Flutter is a condition of instability caused by interaction between a structural system immersed in a flowing fluid at certain speed. The flutter phenomena can lead to catastrophic failure of the structure and for this reason is extensively studied. The common practice in industry is perform flutter stability analyses considering the generalized stiffness and mass matrices obtained from Finite Element method (FEM) and aerodynamic generalized forces from a panel method, as the Doublet Lattice method. These analyses are often re-performed if significant differences are found in structural frequencies obtained from ground vibration tests (GVT) compared with FEM. This unavoidable rework results in a lengthy and costly process of analysis during the aircraft development. In this context, this paper presents a flutter analysis considering structural frequencies uncertain. The aeroelastic system is written as an Affine Parameter model and the robust stability is verified solving a Lyapunov function through Linear Matrix inequalities and convex optimization.

Palavras-chave: Flutter, Linear Matrix Inequalities, Structural Uncertainties

1 INTRODUCTION

Flutter is a condition of instability caused by interaction between a structural system immersed in a fluid at certain speed. This phenomenon can lead to catastrophic failure of the structure and for this reason is extensively studied in aeroelasticity [1].

Different approaches have been proposed to identify the flutter boundaries. In general, the methods are formulated in frequency domain as eigenvalue problems for which the aeroelastic model is defined for each point in the flight envelope (a pair of altitude/air density and velocity) [2], [3], [4]. This can be costly in terms of computational time and engineering analysis of data if a large number of points need to be calculate. Also, the uncertainties in the aircraft model makes the prediction of the stability boundary difficult [5].

The common practice in aeronautical industry is perform flutter analysis considering the generalized stiffness and mass matrices obtained from FEM and aerodynamic generalized forces from a panel method. Particularly, the structural models are improved using experimental data obtained from GVT (Ground Vibration Test). However, there are serious difficulties to apply this procedure due to the limited availability of the aircraft and the fact that multiple configurations need to be tested, as discussed in reference [6]. Additionally, GVTs of aircraft are typically performed very late in the development process. In fact, flutter analysis is often re-performed if significant differences are found in structural frequencies and damping ratios obtained from GVT compared to FEM and an extreme time pressure exists to get the final results.
In this context, this paper presents an approach to perform flutter analysis including uncertainties in structural frequencies. The main goal is to assure the nominal system stability considering this modal parameter varying in a limited range previously assumed by experience. The aeroelastic system is written as an Affine Parameter model and the robust stability is verified solving a Lyapunov function through LMI (Linear Matrix Inequalities). The method is written in time domain using a rational function approximation for the aerodynamic forces. This methodology offers promise for robust flutter analysis using convex optimization.

2 MATHEMATICAL MODEL

Classical flutter analysis is performed considering a second order model in generalized coordinates. This is a fundamental approach mainly to analyze large and complex structures. The system of equations is projected on the structural eigenvector obtained without structural damping. Then, the equation of motion is represented in Laplace domain as

\[ s^2 M_m u_m(s) + s D_m u_m(s) + K_m u_m(s) = q Q_m(m_M, k) u_m(s) \]  

where the subscript \( m \) indicates the generalized domain, \( q = \frac{1}{2} \rho V^2 \) is the dynamic pressure, \( \rho \) is the air density and \( V \) is the airspeed. The matrix \( M_m \) is an identity matrix (using the eigenvector normalized by the modal mass) and the stiffness and damping matrices are both diagonal matrices respectively given by

\[ K_m(x, x) = \lambda x = \omega_x^2 \]
\[ D_m(x, x) = 2 \xi_x \omega_x, \quad x = 1, ..., m \]  

\( Q_m \) is a matrix of aerodynamic coefficients that is a function of the Mach Number and reduced frequency \( k \) and considered time invariant. Because this matrix has no Inverse of Laplace Transform, a rational function approximation is used to write the aerodynamic forces in time domain [9]. In this case, Eq. (3) contains a polynomial part representing the forces on the section acting directly connected to the displacements \( u_m(t) \) and their first and second derivatives. Also, this equation has a rational part representing the influence of the wake acting on the section with a time delay.

\[ Q_m(s) = \sum_{j=0}^{2} Q_{mj} s^j \left( \frac{b}{V} \right)^j + \sum_{j=1}^{n_{lag}} Q_{m(j+2)} \left( \frac{s}{s + \frac{b}{V} \beta_j} \right) u_m(s) \]  

where \( s \) is the Laplace variable, \( n_{lag} \) is the number of lag terms and \( \beta_j \) is the \( j \)th lag parameter \((j = 1, \ldots, n_{lag})\).

The proposed approach is formulated specially for system which small variations in frequencies due to uncertainties of structural properties and modelling have not substantial impact on their structural modes. This assumption is commonly used by researchers in different methods [7] and [8]. It was considered valid for the example shown in this work and, mainly for complex structures, it can be previously confirmed through numerical tests.

To formulate the problem including the uncertainties an affine parameter model is used. An affine parameter model is a special state space equation which some constant uncertain parameter have a fixed value that is known only approximately. Gahinet and his colleague [10] discuss details and advantages to use this approach and, in practice, the equation of motion is written as

\[ E(\theta) \dot{x}(t) = A_E(\theta) x(t) \]  

where \( E(\theta) \) and \( A(\theta) \) are known matrices written as functions of the vector \( \theta = (\theta_1, ..., \theta_{n_\theta}) \in \mathbb{R}^{n_\theta} \) of real uncertain parameters \( \theta_x \in [\theta_x^{min}, \theta_x^{max}], \quad x = 1, ..., n_\theta \); and

\[ A_E = A_0 + \theta_1 A_{\theta_1} + ... + \theta_{n_\theta} A_{\theta_{n_\theta}} \]
In this work the robust aeroelastic quadratic stability is verified considering that \( \bar{m} \) structural frequencies are uncertain and can vary in a limited range, as shown in the following equations

\[
\begin{align*}
\omega_{x}^{\text{unc}} &= \omega_{x} + \Delta \omega_{x} \\
\xi_{x}^{\text{unc}} &= \xi_{x} + \Delta \xi_{x} \\
x &= 1, \ldots, \bar{m} \leq m
\end{align*}
\]  

(6)

where \( \omega_{x} \) and \( \xi_{x} \) are their nominal values computed by FEM.

### 3 Robust Stability Analysis

Linear matrix inequalities (LMI) have been extensively applied in modern control theory [11]. LMI contributed to overcome many difficulties in control design. In the last decade, LMIs have been used to solve many problems that until then was unfeasible through others methodologies, due mainly to the emerging of powerful algorithms to solve convex optimization problem, as for instance, the interior point method [11] and [10]. There are few studies involving LMIs to solve problem in aeroelastic fields and this work introduces an extension of the ideas previously discussed in references [12].

Based on LMI, a sufficient condition for the quadratic stability of Affine system represented by \( \dot{x}(t) = E_{0}^{-1}A(\theta) \) is the existence of \( \bar{m} + 1 \) symmetric matrices \( P_{i} \) such that [10]

\[
\begin{align*}
A(\theta)^{T}P(\theta) + P(\theta)A(\theta) &< 0, \quad \forall \theta \in \nu \\
P(\theta) &> I, \quad \forall \theta \in \nu \\
A_{i}^{T}P_{i} + P_{i}A_{i} &> 0, \quad \forall i = 1, \ldots, \bar{m} \in \nu
\end{align*}
\]  

(7)

where \( \nu \) denotes a set of vertices of a hyperrectangle and \( P(\theta) := P_{0} + \theta_{1}P_{1} + \ldots + \theta_{m}P_{m} \). Gahinet et al. [10] show if this LMI system is feasible, the quadratic stability is assured for all values of \( \theta_{x} \) in \( \theta_{x}^{\text{min}}, \theta_{x}^{\text{max}} \), where \( \theta_{x}^{\text{min}} = \theta_{x}(1 - \Theta \delta \theta_{x}) \) and \( \theta_{x}^{\text{max}} = \theta_{x}(1 + \Theta \delta \theta_{x}) \), \( \forall x = 1, \ldots, \bar{m} \). Note that \( E_{0} \) is a non-singular matrix; otherwise see reference [13].

Since the quadratic stability is a sufficient condition, this approach computes the range which the parameters can vary keeping the system stable. However, it does not necessarily compute the largest variation that is possible.

#### 3.1 Structural Frequencies Uncertainties

Consider an undamped system (\( D_{m} \) is a null matrix) with uncertainties in its \( \bar{m} \) elastic frequencies \( \omega_{x}, x = 1, \ldots, \bar{m} \). Since each \( x \)th frequency is written as its square into the generalized stiffness matrix, the approach is mathematically formulated considering the parameter \( \theta_{x}^{\text{unc}} = (\omega_{x}^{2})^{\text{unc}} \). This notation allows to write the aeroelastic matrix as shown bellow

\[
A_{E} = A_{0} + (\omega_{1}^{2})^{\text{unc}}A_{\omega_{1}} + \ldots + (\omega_{\bar{m}}^{2})^{\text{unc}}A_{\omega_{\bar{m}}}
\]  

(8)

and the system described by Eq. (4) is written such that

\[
E_{0} = \begin{bmatrix}
M_{am} & 0 & \cdots & 0 \\
0 & I & 0 & \cdots \\
0 & \cdots & \ddots & \cdots \\
0 & \cdots & 0 & I
\end{bmatrix}
\]  

(9)

\[
M_{am} = M_{m} - q \left( \frac{b}{V} \right)^{2} Q_{m2}
\]  

(10)
where $\bar{K}_m$ is the modal stiffness matrix setting zero for each $x$th frequency considered uncertain. For each matrix $A_{\omega_x} \in m(2 + n_{lag}) \times m(2 + n_{lag})$ in Eq. (5), the element $a_{(x,x+m)} = -1$ and the other ones are zero. Note that each of these matrices have only one non-zero element and each $E_{\omega_x}$ are null matrices. The uncertain parameter $(\omega_{x}^{unc})^2$ is defined in a continuous range through its minimum and maximum values according to the following equation

$$(\omega_{x}^{unc})^2 = \omega_x^2(1 \pm \delta_{\omega_x}) \quad \text{such that,}$$

$$(\omega_{x}^{unc})^2_{min} = \omega_x^2(1 - \delta_{\omega_x})$$

$$(\omega_{x}^{unc})^2_{max} = \omega_x^2(1 + \delta_{\omega_x})$$

$$x = 1, \ldots, \bar{m}$$

where $\delta_{\omega_x}$ is the percentage of uncertainty in the $x$th eigenvalue $\lambda_x$ (or $\omega_x^2$). It is possible to rewrite Eq. (12) introducing the parameter $\Theta > 0$ for computing the largest portion of the specified parameter range $\theta_x \in [\theta_{x}^{min}, \theta_{x}^{max}]$ where quadratic stability is assured, as shown bellow

$$(\omega_{x}^{unc})^2_{min} = \omega_x^2(1 - \Theta \delta_{x})$$

$$(\omega_{x}^{unc})^2_{max} = \omega_x^2(1 + \Theta \delta_{x})$$

In this case, the aeroelastic quadratic stability is assured considering that each $x$th structural frequency can vary $\tau_{\omega_x}$ percent with respect to its nominal value, where

$$\tau_{\omega_x} = 10^2 \left[\left(1 + \Theta \delta_{\omega_x}\right)^{1/2} - 1\right]$$

(14)

Note that $\omega_{x}^{unc} = \omega_x[1 \pm 10^{-2} \tau_{\omega_x}]$; and $\delta_{\omega_x}$ is previously defined to solve an LMI system and $\Theta$ is an output.

### 4 NUMERICAL APPLICATION

To illustrate the effectiveness of the method, numerical simulations were developed on a benchmark wing structure AGARD 445.6 shown in reference [14]. The linear structural model for the AGARD 445.6 wing was created using the MSC/NASTRAN program. The wing is modelled with plate elements as a single layer orthotropic material consisting of 231 nodes and 200 elements. The thickness distribution was governed by the airfoil shape. The material properties used are $E_1 = 3.1511$ GPa, $E_2 = 0.4162$ GPa, $\nu = 0.31$, $G = 0.4392$ GPa and $\rho_{mat} = 381.98$ kg/m$^3$, where $E_1$ and $E_2$ are the moduli of elasticity in the longitudinal and lateral directions respectively, the $\nu$ is Poisson’s ratio, $G$ is the shear modulus in each plane and $\rho_{mat}$ is the mass density. Small damping loss factor was considered using $\xi = .01$ for all natural frequencies.

Aerodynamic and structural matrices are obtained by MSC/NASTRAN program (from solution 145) for Mach number 0.50 and $\rho_{REF} = 1.225$ kg/m$^3$ (air density). Figure 1 presents the flight envelope considered to illustrate this application showing the discrete points where the analysis was performed. This figure also shows the boundaries of stability of flutter. The values of reduced frequencies are $10^{-3}$, $2.10^{-3}$, $5.10^{-3}$, $10^{-2}$, $5.10^{-2}$, 0.1, 0.2, 0.3, 0.5, 0.6, 0.8, 1.0, 1.5, 2.0, 3.0 and 4.0. The model has a length of reference $2b = 0.5578$ m, a sweep angle is 45 degrees at the quarter chord line, a semi-span of 0.762 m and a taper ratio of 0.66. The flutter boundary is investigated using the first four fundamental structural modes ($m = 4$). Their natural frequencies ($V = 0$) are $\omega_1 = 9.45$ Hz, $\omega_2 = 39.69$ Hz, $\omega_3 = 49.45$ Hz and $\omega_4 = 95.10$ Hz. A state
space model in time domain was obtained using four parameters of lag $\beta_1 = 0.55$, $\beta_2 = 1.40$, $\beta_3 = 1.90$ and $\beta_4 = 2.90$.

To apply the approach is verified that the structural modes are not significantly affected by the uncertainties considered during the analysis. This is an assumption to assure the results specially for systems represented by equation of motion written in the generalized coordinate system with modal truncation, however, it is not required for small systems written in the physical coordinate system. For the system presented in this work that behavior was verified by computing the modal assurance criterion (MAC) using the structural eigenvectors obtained from different values of longitudinal modulus of elasticity. The particular example shown in Fig. 2 demonstrates that two sets of eigenvectors are consistent when the structural frequencies are not their nominal values (see Table 1). The MAC is a index that shows how the eigenvectors correlate, for perfect correlation, one would obtain the identity matrix, and in this case, although there are difference in the material properties and variation in the natural frequencies, the eigenvectors are still very similar..

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>$E_1$ (nominal)</th>
<th>$E_1 = 2.50$GPa</th>
<th>Variation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.45</td>
<td>8.49</td>
<td>-10.19</td>
</tr>
<tr>
<td>2</td>
<td>39.69</td>
<td>37.15</td>
<td>-6.41</td>
</tr>
<tr>
<td>3</td>
<td>19.45</td>
<td>47.04</td>
<td>-4.87</td>
</tr>
<tr>
<td>4</td>
<td>95.10</td>
<td>89.33</td>
<td>-6.06</td>
</tr>
</tbody>
</table>

Let the first structural frequency uncertain ($\bar{m} = 1$). Using the pk-method [15] the flutter speed was computed considering the nominal value of $\omega_1$ ($V_F = 158.02$ m/s; see the Vg plot in Fig. 3). This figure shows that becomes unstable when one of the eigenvalues are positive. The LMI system shown in Eq.s (7) was solved considering $\delta_1 = 0.2$. According to the proposed approach, Fig. 4 shows the range which the first structural frequency can vary keeping the system stable. The stability was confirmed by performing the pk-method in the limits of the range using $\lambda^{unc}_1 = (1 + \delta_1)\omega^2_1$ and $(1 - \delta_1)\omega^2_1$ into the stiffness matrix.
Figure 2: A three-dimensional presentation of MAC values.

Figure 3: Nominal Vg plot obtained from pk-method.

References


Figure 4: Index $\tau$ for the first structural frequency uncertain.


