Estimation of Monetary Policy Preferences by a Genetic Algorithm

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Abstract: In this article, we estimate the preferences that have guided the Central Bank of Brazil in conducting the monetary policy. Use of genetic algorithm to determine the preferences is proposed as an alternative to the usual grid-search method. The results suggest that the Central Bank has adopted a flexible inflation targeting regime.

Keywords: Monetary Preferences of the Central Bank, Optimal Monetary Policy, Linear-Quadratic Regulator Problem.

1. INTRODUCTION

Since June 1999, Brazil has adopted the inflation targeting regime as its monetary policy guideline. In this regime, the central bank announces to the public a mid-term target for inflation and undertakes to act so that the actual inflation moves towards that goal. The main instrument for achieving this is the SELIC interest rate, adjusted at regular meetings of the Monetary Policy Committee (Copom) [1].

While monetary policy in Brazil and in the world arises from a complex procedure involving a large amount of subjective evaluations, it is possible to model the decision making process as an optimal control problem. This is done by assuming that the Central Bank's goal is to minimize a quadratic loss function, restricted by a linear model that represents the connections between macroeconomic variables. Thus, monetary policy is posed as a problem of linear-quadratic regulator (LQR), where the Central Bank preferences can be associated with the weights that define the loss function [2].

Considering this approach, the identification of preferences of Central Bank has emerged as a key point for a deeper understanding of monetary policy. According to Dennis [3], knowledge of these preferences, besides contributing to the understanding of monetary policy actions, allows for a comparison of the goals of Central Bank in different intervals of time. In this framework, there is an extensive literature devoted to estimating the weights that the monetary authority assigns to the variables in their loss function. However, most of these studies have focused on the preferences of the Federal Reserve (FED) and Central Banks from European countries (e.g. [4,5]).

With regard to monetary policy in Brazil, only Aragon and Portugal [6] have presented estimates for the preferences of the Central Bank of Brazil. Following [7, 8, 9], these authors applied a grid-search method to calibrate the loss function of the central bank, choosing the weights such that the solution of the corresponding optimal control problem resulted in the best fit between the real monetary policy and the simulated optimal monetary policy.

2. PURPOSE

Motivated by the scarcity of studies of this kind on brazilian monetary policy, this article aims at estimating the preferences of the Central Bank of Brazil during Henrique Meirelles's mandate as chairman. For this, the calibration strategy is also used. Use of genetic algorithm with real coding to determine the preferences is proposed as an alternative to the usual grid-search method.

3. METHODS

3.1. Macroeconomic Model

The macroeconomic model used in this work is inspired by that proposed by Rudebush and Svensson [10]. The equations that constitute the model are:

\[ \pi_{t+1} = \alpha(L)\pi_t + \alpha(L)\pi_t' + \alpha_i(L)\beta_t + \alpha_c(L)\Delta \pi_t + \epsilon_{\pi,j} , \]

\[ h_{t+1} = \beta_\delta(L)h_t + \beta_\gamma(L)(i_t - \pi_t') + \beta_c(L)c_t + \epsilon_{h,j} , \]

\[ \sigma_{t+1} = \gamma_{\sigma}(L)\sigma_t + \gamma_c(L)\Delta \sigma_t + \gamma_i(L)\Delta \sigma_t + \epsilon_{\sigma,j} , \]

\[ c_{t+1} = \delta_{\sigma}(L)c_t + \delta_\sigma(L)\pi_t + \delta_c(L)\pi_t' + \delta_{\pi}(L)c_t + \epsilon_{c,j} , \]

where,

\[ \alpha(L), \beta(L), \epsilon_t, \] are polynomials in the lag operator \( L \),

\[ Lx = x_{t-1} ; \]

\( \pi_t \) corresponds to the inflation rate (IPCA) accumulated in 12 months in the t period: \( \ln(1 + IPCA_t / 100) \);

\( \pi_t' \) corresponds to the expected inflation rate in t period for the next 12 months: \( \ln(1 + IPCA_t / 100) \);

\( h_t \) corresponds to the output gap in t period, estimated by Hodrick-Prescott filter: \( \ln(1 + Gap_t / 100) \);

\( c_t \) corresponds to the real effective exchange rate in t period: \( \ln(1 + Exchange\_Rate_t) \), \( \Delta c_t = c_t - c_{t-1} \);

\( i_t \) corresponds to the SELIC interest rate prevailing at end of t period: \( \ln(1 + SELIC_t / 100) \), \( \Delta i_t = i_t - i_{t-1} \).
The random errors occurring in the t period. To estimate the coefficients of the model, the Ordinary Least Square (OLS) method was applied to each equation separately, considering quarterly data from the third quarter of 2001 (2001/T3) until the second quarter of 2010 (2010/T2), obtained from the websites of Central Bank of Brazil and IPEADATA. In order to avoid outliers, we included binary variables in the model for presidential succession in 2003 [2002/T4 - 2003/T1] and to account for the international financial crisis [2008/T4 - 2009/T1].

As the above equations involve polynomials in the lag operator L, we had to choose which lags to include in the model. First, assuming a maximum of three lags for the variables, we estimate all possible combinations for each equation. So, among the specifications thus obtained, we chose the one that resulted in the lowest value of the Schwarz Information Criteria (SIC). The model equations thus obtained (without the binary variables and errors) read as follows:

\[
\begin{align*}
\pi_{t+1} & = 0.716\pi_t - 0.267\pi_{t-1} + 0.609\pi_{t-2} + 0.154\pi_t - 0.037\Delta c_t, \\
h_{t+1} & = 0.827h_t - 0.152(i_t - \pi_t^*) + 0.0038c_t, \\
\pi_{t+1} & = 1.048\pi_t^* + 0.098\pi_{t-1} - 0.185\pi_t - 0.024\Delta c_t - 0.222\Delta i_t, \\
c_{t+1} & = 0.982c_t - 0.940\pi_t + 1.983\pi_t^* - 1.759\Delta i_t.
\end{align*}
\]

### 3.2. Optimal Control Problem of Central Bank

Based on [2], it is reasonable to assume that the choice of the SELIC rate is guided by the following objectives: (1) keeping inflation around the target \( \pi^* \); (2) to promote economic stability; and (3) make gradual adjustments to the monetary policy preferences. As such, the trajectory of the SELIC rate \( (i_t) \) that best meets the goals (1) - (3) is that which minimizes current and future Central Bank’s loss. Therefore monetary policy can be viewed as the following optimal control problem:

\[
\begin{align*}
\text{Minimize } & E_0 \left[ \sum_{t=0}^{\infty} \delta^t \left( \pi_t - \pi^* \right)^2 + \lambda_h h_t^2 + \lambda_i (i_t - i_{t-1})^2 \right] \\
\text{Subject to the Macroeconomic Model.}
\end{align*}
\]

This is the problem of minimizing the expected loss over an infinite horizon. To solve it, we use Dynamic Programming. The associated Bellman equation is given by:

\[
V(X_t) = \min_{\{u_t\}} \left[ \delta^t P_t X_t + \delta(1 - \delta)^{t-1} \mathbb{E} \left[ V(X_{t+1}) \right] \right],
\]

where \( V(X_t) \) is the value function, \( P_t \) is the transition matrix, \( \delta \) is the discount factor, and \( \mathbb{E} \) denotes expectation. The optimal policy is obtained by backward induction, starting from the last period and working backwards to the current period.

### 3.3. Calibration Problem of Monetary Preferences

Formally, the calibration of the loss function is equivalent to identifying the monetary preferences as the solution of the following optimization problem [7,8]:

\[
\begin{align*}
\text{Minimize } & \frac{1}{T} \left[ \sum_{t=1}^{T} \left( i_t - i_t^* \right)^2 \right] \\
\text{Subject to } & \left( \delta, \lambda_{x}, \lambda_{h}, \lambda_{i} \right) \in \Omega,
\end{align*}
\]

where \( i_t \) is the real value of SELIC rate and \( i_t^* (\delta, \lambda_{x}, \lambda_{h}, \lambda_{i}) \) is the value of SELIC rate given by the optimal monetary policy rule \( i_t = -F(\delta, \lambda_{x}, \lambda_{h}, \lambda_{i}) X_t \). Because of the peculiar nature of this problem, in the papers cited above sub-optimal solutions are obtained by applying a grid-search method. First, \( \Omega \) space is discretized in order to narrow the search to a finite subset \( S \subseteq \Omega \). Then, for all elements \( \left( \delta, \lambda_{x}, \lambda_{h}, \lambda_{i} \right) \in S \), the corresponding LQR problem is solved, resulting in optimal rules of type \( i_t = -F(\delta, \lambda_{x}, \lambda_{h}, \lambda_{i}) X_t \). Replacing the actual values of \( X_t \),

\[
\begin{align*}
\varepsilon_{x,t}, \varepsilon_{h,t}, \varepsilon_{i,t},
\end{align*}
\]
they calculate the optimal path of SELIC rate associated with each element in $S$. Finally, among all vectors $(\delta, \lambda_x, \lambda_h, \lambda_i)$ considered, the monetary policy preferences are represented by one that implies the smallest mean-square deviation:

$$MSD(i, i_t) = \frac{1}{T} \sum_{t=1}^{T} [i_t - i_t(\delta, \lambda_x, \lambda_h, \lambda_i)]^2$$

3.4. Genetic Algorithm

Aiming to obtain better estimates, this paper proposes the use of a genetic algorithm with real coding to solve the problem of calibration of monetary preferences. Genetic Algorithms (GAs) are stochastic optimization methods inspired by natural selection and genetics. In this type of method, the search begin by a population of possible solutions, called individuals. Once represented by chromosomes, individuals are subjected to three basic operators: selection, crossover and mutation. The fittest chromosomes, individuals are chosen to generate the new population. Then, the crossover combines the features of two pre-selected individuals to get fitter individuals. Finally, the mutation changes the individuals randomly, preserving diversity in the population. This process is repeated a number of times ($k_{max}$), and the final fittest individual is taken as solution of the problem. The flowchart illustrates the algorithm in Figure 1.

- Fitness: To Solve the problem (5)-(6), the fitness of each $(\delta, \lambda_x, \lambda_h, \lambda_i) \in \Omega$ was defined by the equation $fitness(\delta, \lambda_x, \lambda_h, \lambda_i) = 1/MSD(i, i_t)$
- Selection: Conducted by Roulette Method;
- Arithmetic Crossover: Two individuals $x, y \in \Omega$ are selected for crossover with probability $p_c = 0.60$. The new individuals follow by the equalities, $x' = ax + (1-a)y$ and $y' = cy + (1-c)x$, where $a$ is a random number with uniform distribution in $[0,1]$.
- Non-Uniform Mutation: Let $x_k \in [a,b]$ be the $k$th coordinate of a vector $x \in \Omega$, selected for the mutation in the $t$ iteration with probability $p_m = 0.3$. Then, sorting a number $s$ in the set $\{0,1\}$, and assuming a maximum number of iterations ($T_{max}$), the value $x_k$ is changed for $x'_k$ according to the equations:

$$x'_k = x_k + (b-x_k)\left[1-\left(\frac{t}{T_{max}}\right)^d\right], \text{ if } s = 0,$$

$$x'_k = x_k + (a-x_k)\left[1-\left(\frac{t}{T_{max}}\right)^d\right], \text{ if } s = 1,$$

where $r$ is a random number with uniform distribution in the range $[0,1]$, and $d$ is the parameter of non-uniformity.

4. RESULTS

Considering the actual values of the SELIC rate during the period in which Henrique Meirelles was the chairman of the Central Bank of Brazil [2003/T1 - 2010/T2], the GA described above was performed six times, each of which is limited by a maximum of 500 iterations. The best solution resulted in $MSD = 1.7764$, and corresponds to the preferences:

$$(\delta, \lambda_x, \lambda_h, \lambda_i) = (0.9991, 5.8007, 0.0016, 10.7573) \quad (7)$$

For comparison, simulating the preferences estimated by Aragon and Portugal [6], the mean-square deviation was found to be significantly higher (5.2411), which can be attributed to differences in modeling, sampling and estimation method.

The rule of optimal monetary policy corresponding to preferences of Eq. (7) is given by:

$$i_t = -0.21\pi_t - 0.25\pi_{t-1} + 2.64\pi_t^{pe} + 0.66\pi_t^{pe} + 1.49h_t + 0.14h_{t-1} - 0.05c_{t-1} - 0.06c_{t-1} + 0.13i_{t-1}.$$

Figure 2 shows that the trajectory of SELIC rate as obtained from the optimal rate model (Red line) fits reasonably well the real trajectory (blue line) practiced by the central bank.
Preferences given by Eq. (7) suggest that the Brazilian monetary authority has practiced a flexible inflation targeting regime with considerable weight to the smoothing of interest rates and a minimal weight for the output gap. However, unlike the work of Aragon and Portugal, the weight for the control of inflation is lower than that given to the smoothing rate, which agrees with the results reported by Dennis for the FED [4] and Lippi and Neri [5] for the Euro Area, indicating that Brazilian Central Bank is highly cautious when it comes to making changes in SELIC rate.

5. DISCUSSION

Using the optimal rule associated with preferences Eq.(7) we have obtained projections for the SELIC rate for the period [2010/T3 - 2011/T4] as displayed in Figure 3. Because these projections result from solving an optimization problem, they are referred to as optimal forecasts.

As shown in Table 1, for the period of available data [2010/T3 - 2011/T1], the optimal forecast for the SELIC rate is found to be better than the average short-term projections made by the TOP 5 of Focus Report, collected throughout the month of July 2010.

Table 1. Performance of the projections for the SELIC interest rate

<table>
<thead>
<tr>
<th>PROJECTION</th>
<th>MRE (%)</th>
<th>RMSE (p.p)</th>
<th>MAE (p.p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPTIMAL</td>
<td>5.95</td>
<td>0.45</td>
<td>0.14</td>
</tr>
<tr>
<td>TOP 5</td>
<td>10.36</td>
<td>1.33</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Although the sample is small, these preliminary results suggest an interesting application of this approach as a forecasting tool for the SELIC interest rate.

5. CONCLUSION

In this article, we estimate the monetary policy preferences of the Central Bank of Brazil during Henrique Meirelles’s period as chairman using a genetic algorithm with real coding. Unlike the results of Aragon and Portugal [6], the weight given to the smoothing of interest rates has been found to be greater than that given to the deviation of inflation, reflecting a very cautious monetary authority. Finally, using the optimal rule associated with the estimated preferences, the forecasts obtained for the SELIC rate are better than those presented in the group of TOP 5 Focus Report, suggesting an interesting application of this theoretical framework to predict the SELIC interest rate.

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