STUDY OF ENERGY DISSIPATION IN BEAMS SUBJECTED TO MOVING LOADS

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Abstract: In this work the vibrations control and energy dissipation of a simply supported beam subjected to moving loads is studied. The beam is controlled by a moving vibration absorber that acts as energy dissipator. The absorber is described as a linear spring-mass-damper system. The results show the importance of the position of the damper on the energy dissipator.

Keywords: energy dissipation, beam oscillations, engineering applications.

1. INTRODUCTION

The study of bridge oscillations and control is a problem that has been the object of interest of engineers and scientist over the last century [1-3]. For example, Den Hartog [4] derived the optimum parameters of the absorber for suppressing the dynamic response of a single degree-of-freedom spring-mass system. Here, only a few investigations will be cited.

Greco and Santini [2], using an extension of the complex mode superposition method, analyzed the dynamic problem of a continuous beam with two end rotational viscous dampers under a single moving load. They concluded that the damper’s effectiveness is strongly dependant on the load velocity and proved that, in the relevant range of velocities, a considerable reduction of the dynamic response of the beam is to be expected if the damper’s constants are selected properly.

Wu [5] proposed the use of helical absorbers to reduce the vibrations of beams subjected to moving loads. To study the behavior of the beam, the governing equations were reduced to the first modal coordinate and, following Den Hartog’s approach [4], this simplified model was used to obtain optimal values for the stiffness and damping ratio of the absorber. The possibility of reduction of the resonant vibration of simple beams under moving loads by increasing the structural damping with passive energy dissipation devices was evaluated by Museros and Martínez-Rodrigo [3]. The authors used a linear viscous damper (FVDs) to connect the main beam, which carries the loads, to an auxiliary beam placed underneath the main one. The results show that the resonant response of the main beam can be drastically reduced with this type of device.

Moreover, the damper attached to the beam can act as passive absorber of vibration energy. Georgiadess and Vakakis [6] showed numerically that an appropriately designed and placed nonlinear mass-spring absorber, designated nonlinear energy sink (NES) can passively absorb and locally dissipate a major portion of the shock energy of the beam, a result that paves the way for the implementation of the NES concept to flexible systems. Recently, Samani and Pellicano [7] analyzed the effectiveness of a dynamic vibration absorber applied to a beam excited by moving loads. The performance of the dynamic dampers in vibration reduction was estimated through the maximum amplitude of vibration and by the amount of energy dissipated by the dynamic damper.

In this work, the Euler-Bernoulli linear beam theory is used to study the vibrations control and energy dissipation of simply supported beams subjected to moving loads and controlled by a fixed or moving absorber. The beam is considered as a linear elastic continuous system and the absorber is described as a linear spring-mass-damper system moving with a specified velocity along the beam. A modal expansion with five modes is used to model the lateral displacements of the beam and the Galerkin method is used to obtain a set of discretized equations of motion which are, in turn, solved by the Runge-Kutta method. The rate of input energy dissipation is obtained by the ratio between the energy absorbed by the damper and internal energy of the beam due to loading. The obtained results show the importance of position and velocity of the damper on the vibration control and in energy dissipation of the beam. This can be used for engineers to optimize the position of vibration absorbers.

2. PROBLEM FORMULATION

Consider a simply supported elastic beam with length L, Young’s modulus E, inertia I, distributed mass m and damping coefficient c subjected to a moving load F(x,t) with velocity V, as shown in Figure 1. The beam is connected to an absorber represented by a small mass ma, a linear spring with stiffness ka and a linear viscous damper with damping coefficient λ. It is assumed that the
The absorber can either be fixed or move along the beam with a velocity $V_A$.

The absorber can passively absorb a major portion of the vibration energy of the beam induced by the load, thus acting as an energy dissipator. Furthermore, the targeted energy transfer from the linear beam to the absorber can be optimized by appropriate design and placement of the absorber [6].

![Figure 1 - The controlled beam model](image)

In this work the mathematical formulation will follow that previously presented by Samani and Pellicano [7]. The partial differential equations of motion governing the flexural behavior of a simply supported beam using the linear Euler-Bernoulli theory and absorber can be found in the works Yang, Yau and Hsu [1], Greco and Santini [2]; and Musserosa and Martinez-Rodrigo [3] and are written as:

\[
EI \frac{\partial^4 y}{\partial x^4} + \frac{m}{\partial t^2} \frac{\partial^2 y}{\partial x^2} + \left[ k_A u + \lambda \frac{\partial u}{\partial t} \right] G(x,t) = F(x,t) \quad (1)
\]

\[
m_A \frac{\partial^2 y}{\partial t^2} - k_A u + \lambda \frac{\partial u}{\partial t} = 0 \quad x \in (0,L) \ t > 0 \quad (2)
\]

where $y(t)$ represents the transversal displacement field of the beam; $u(t)$ is the absolute position of the mass $m_A$; $d$ is the absorber position and $u(t) = y(d, t)-v(t)$.

In Eq. (1) the term $\left[ k_A u + \lambda \frac{\partial u}{\partial t} \right] G(x,t)$ is the force exerted by the absorber and $G(x,t)$ is given by:

\[
G(x,t) = \delta(x-d) \quad \text{For a fixed absorber;} \quad (3)
\]

\[
G(x,t) = \delta(x-V_d t)H \left( \frac{L}{V_A} - t \right) \quad \text{For a moving absorber;} \quad (4)
\]

where $x = d$ represents the location of the damper in the beam at time $t$; $V_A$ is the absorber velocity; $\delta$ is the Dirac delta function which defines the location of the dynamic damper while $H(t)$ is the Heaviside function.

The external force $F(x,t)$ is a moving load given by:

\[
F(x,t) = F_o \delta(x-V_d t)H \left( \frac{L}{V_A} - t \right). \quad (5)
\]

The attached mass is small compared to the beam mass. In this work, the lumped mass of the absorber is taken to be 5% of the total mass of the beam [5]. The equations of motion of the system, represented by Eqs. (1) and (2), are analyzed after projecting the partial differential Eq. (1) into a complete orthonormal basis [7].

The eigenfunctions of the simply supported beam with no attachments can be used as interpolating functions. They are given by:

\[
\phi_r(x) = \sin \left( \frac{r \pi x}{L} \right) \quad r = 1, 2, 3, \ldots \quad (6)
\]

The natural frequency of the beam for the $r^{th}$ mode is given by $\omega_r = (\pi r)^2 \sqrt{EI/\rho I L^3}$ and the transverse vibration of the beam can be assumed as [3, 7]:

\[
y(x,t) = \sum_{r=1}^{\infty} A_r(t) \phi_r(x) \quad (7)
\]

where $A_r(t)$ are the unknown functions of time and $\phi_r(x)$ is given by Eq. (6).

Substituting Eq. (7) into Eqs. (1) and (2), applying the Galerkin method and using the orthonormality conditions, the following system of equations is obtained:

\[
\frac{m_L}{2} \ddot{\Lambda}_p(t) + \xi_p \omega_p m_L \Lambda_p(t) + \frac{\omega_p^2 m L}{2} A_p(t) +
\]

\[
+ \left[ k_2 \sum_{r=1}^{\infty} A_r(t) \phi_r(D) - v(t) \right] \phi_p(D) G(t) = 0 \quad (8)
\]

\[
F_o \phi_p(V_L t) H \left( \frac{L}{V_L} - t \right) \quad (9)
\]

where $\dot{A}_p(t) = dA_p / dt$; $D = d$ and $G(t) = 1$ for fixed absorber; $D = V_d t$ and $G(t) = H(1/V_A - t)$ for moving absorber.

The portion of the input energy dissipated by the viscous damper at time $t_i$ is computed by the expression [6, 7]:

\[
\eta = \frac{E_{ED}}{E_{IN}} = \frac{\int_0^{t_i} \dot{\Lambda}_p(t) - \sum_{r=1}^{n} \dot{A}_r(t) \phi_r(D) \phi_p(V_L t) dt}{\int_0^{t_i} F_o \phi_p(V_L t) dt} \quad (10)
\]

$E_{ED}$ is the energy passively absorbed and locally dissipated by the absorber, $E_{IN}$ represents the total energy input of the beam due to the load and $t_0$ is the load duration in the beam.

### 3. NUMERICAL RESULTS

For the numerical analysis, consider a beam with Young’s modulus $E = 206.8$ GPa, mass density $\rho = 7820$ Kg/m$^3$, cross-sectional area 0.03 m x 0.03 m and $F_o = 9.8$ N. Three different lengths are considered say beam L4, L5 and L6 and are chosen only to extend the work by Samani and Pellicano [7]. They are shown in Table 1 together with the associated system parameters.
3.1. Fixed absorber and moving load

To check the accuracy of the present model, consider the system shown in Figure 1 with a fixed absorber \((V_0(t) = 0\) and \(d = 0.5L)\) and external force with constant amplitude \((F_c)\) and constant velocity \(V_L(t) \neq 0\). Figure 2 shows the maximum displacement at the mid-span of the \(L4\) beam with and without damper and increasing values of load velocity. As can be observed, for this beam the maximum displacement occurs for a velocity \(V_L = 21.2\ m/s\). The inclusion of the absorber reduces the maximum displacement of the beam up to 5%.

![Figure 2 - Maximum displacement at mid-span of the beam (\(x = 0.5L\)). Beam \(L4\) with and without absorber.](image)

The optimization of the dynamic damper is focused on the minimization of the maximum beam displacement. Then, the location of the dynamic absorber can be varied to find the optimum absorber position. For each beam, the absorber position is varied considering the load velocities \(V_L\) that generates the maximum displacement at the beam. The obtained results are displayed in Table 2 where the ratio \(l_{mid}/L\) indicates the position at which the maximum displacement occurs and \(d/L\) indicates the best position of absorber.

![Table 1 – Parameters of the beam and absorber with \(\mu = 0.1\;
\alpha_0 = 0.909 and \(\xi_{stat} = 0.18464\).](image)

<table>
<thead>
<tr>
<th>L (m)</th>
<th>(\bar{m}) (Kg/m)</th>
<th>(\omega_0) (rad/s)</th>
<th>(m_s) (Kg)</th>
<th>(k_A) (N/m)</th>
<th>(A) (Ns/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>7.038</td>
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<td>1.4076</td>
<td>877.92</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>12.209</td>
<td>2.1114</td>
<td>286.11</td>
<td>9.076</td>
</tr>
</tbody>
</table>

3.2. Moving absorber and moving load

Figure 3 shows the fraction of energy dissipated by the viscous damper, \(\eta\), for different absorber positions for \(t_1 = 1\ s\) and \(t_2 = 30\ s\) and two different damping ratios of the principal system. As can be observed in both Table 2 and Figure 3, the best absorber position is located close to the mid-span of the beam.

![Table 2 – Location of maximum beam displacement and optimal absorber position.](image)

<table>
<thead>
<tr>
<th>(V_L) (m/s)</th>
<th>(l_{mid}/L)</th>
<th>(d/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L4</td>
<td>21.2</td>
<td>0.525</td>
</tr>
<tr>
<td>L5</td>
<td>16.7</td>
<td>0.520</td>
</tr>
<tr>
<td>L6</td>
<td>13.9</td>
<td>0.533</td>
</tr>
</tbody>
</table>

4. CONCLUSION

In this work the control vibration of simply supported beams subjected to moving loads was studied. Both, fixed and moving absorber were considered, for this the linear Euler-Bernoulli beam theory was used together with the Galerkin method to obtain a set of ordinary differential equations of dynamic equilibrium.

For fixed absorber, the obtained results show the influence of load velocity and absorber position on the reduction of the maximum beam displacement. Results show that, when the absorber position is close to the beam mid-span, there is a maximum transversal displacement reduction.

For moving absorber, it is possible to observe that the maximum reduction is obtained when the ratio between load and absorber velocity is close to one and the maximum energy dissipated is obtained for low \(V_0/V_L\) ratios which correspond to the load velocity that generates maximum displacement.
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REFERENCES


