NONLINEAR NORMAL MODES OF COUPLED SELF-EXCITED OSCILLATORS

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Abstract: The main purpose of this paper is to present nonlinear modes formulation which can be applied to a class of coupled self-excited nonlinear oscillatory systems under selected resonance conditions. A many degree of freedom system is transformed into the problem with reduced number of degrees by application of nonlinear normal modes. The modes are formulated with respect to the system’s vibration amplitude on the basis of free vibrations of a nonlinear conservative system and then applied to systems with force or parametric excitations. The analysis shows effectiveness of the method for three examples: classical viscous damped oscillator and for self-excited models with parametric and external excitations. An important difference between linear and nonlinear mode formulation is presented for selected resonance conditions.

Keywords: Vibrations, Nonlinear Normal Modes, Self-Excitation.

1. INTRODUCTION

Interaction between different vibration types may lead to very interesting and untypical phenomena. Due to nonlinear damping and nonlinear elasticity characteristic the system can exhibit untypical and intuitively unexpected phenomena. If the system is additionally excited by periodic force or/and excited parametrically then additional interactions near the resonance neighbourhoods occur. Even for the simplest one degree of freedom model many new dynamical phenomena appear [1], [2].

Interest of this paper, in particular, is to determine Nonlinear Normal Modes of a coupled many degree of freedom system composed of nonlinear self-excited oscillators. The formulation should allow for reduction of the problem without any loss of dynamics of the original system.

Due to nonlinear damping, producing self-excitation, the vibrations modes are strongly, both, displacement (amplitude) and velocity dependent. Therefore a transformation by linear normal modes does not guarantee a proper decoupling of the model. Moreover, if the self-excited oscillators are driven by parametrically or by external forces, their response can be periodic, quasi-periodic or chaotic. For the one degree of freedom system the regions of mentioned vibration types can be found by analytical approximate methods or directly by numerical methods. The problem arises if the model has many degrees of freedom. For nonlinear systems the Linear Normal Modes (LNMs) can be used by making assumption that the model is weakly nonlinear. However, for strong nonlinearities that approach leads to results which are far from real dynamics. For driven parametrically and self-excited systems two possible vibrations modes may appear: (a) resonant modes, inside the parametric resonance region where single frequency response takes place, (b) non-resonant modes, manifested by quasi-periodic response. Analysis of the resonant modes has been presented in the papers [3], [5], [6] while the nonlinear modes of coupled self-excited oscillators (an autonomous model) without internal resonance condition have been investigated in [4]. Despite strong nonlinear damping, the system’s motion has been decoupled successfully into two independent modal self-excited oscillators by the centre manifold reduction [8]. The present paper is a review of the former analysis. The nonlinear normal modes (NNM) of coupled self-excited oscillators under resonance conditions are formulated for selected resonance regions. The differences between linear and nonlinear normal modes formulation for selected models and parameters are presented.

2. MODEL OF THE VIBRATING SYSTEM

Let us consider the model of coupled nonlinear oscillators with dynamics represented by a set of differential equations

\[ m\ddot{X} + kX + f_d(X, \dot{X}) + f_p(X, \Omega\tau) = f_e(\omega\tau) \] (1)

where \( X \) and \( \tau \) are dimensionless coordinate and time respectively, \( m, k \) are mass and stiffness matrices related to the linear part, \( f_d(X, \dot{X}), f_p(X, \Omega\tau), f_e(\omega\tau) \) represent nonlinear damping functions, parametric and external excitations. Equation (1) is written in a general form which include nonlinear terms dependent on displacement and velocity as well as time dependent terms. All nonlinear functions couple the system’s vibration modes. A linear decoupling procedure
can be applied only if the system is weakly nonlinear or linear with small damping. It would be rather difficult to propose a method enables decoupling of the system and solving the problem in general. Therefore, Eq. (1) is considered only for selected cases. The case of autonomous nonlinear self-excited system has been studied in [4].

In the present paper we demonstrate a method which can be applied for nonlinear non-autonomous systems near resonance regions. The first considered non-autonomous systems is a classical nonlinear Duffing oscillator with weak viscous damping and then the same technique is applied for self-excited oscillators forced parametrically, and next parametrically and externally.

Differential equation (1) can be transformed in a classical way by linear coordinate transformation

\[ X = uY \]  

where \( u \) is a linear modal vector and \( Y \) vector of normal coordinates. Substituting (2) into (1) and next multiplying both equation sides by \( u^T \) we get

\[ MY + KY = \varepsilon \left[ \hat{F}(Y, \dot{Y}) + \hat{Q}(\omega_0 \tau) \right] \]  

where: \( M = \text{diag}[M_j] = u^T m u, K = \text{diag}[K_j] = u^T k u, \) and \( \omega_{0j} \), means \( j \) frequency of a linear model. A small parameter \( \varepsilon \) is used formally for collecting all nonlinear terms, damping in functions \( \hat{F}, \hat{Q} \), and excitation in \( \hat{Q} \). If the parameter \( \varepsilon \) is equal to zero, then we get a set of uncoupled linear equations which can be solved independently. However, in our case the system is coupled and the nonlinear normal modes have to be formulated bearing in mind system’s nonlinear features.

3. Resonant Nonlinear Normal Modes Formulation

In a classical approach we can assume that vibration modes are close to the linear model and then we can solve only one from a set of equations (3), independently finding \( Y_k = Y_k(t) \) and putting \( Y_j = 0, j = 1, \ldots, n, j \neq k \). The methodology is based on the assumption that near the resonance zones the response corresponds to one “almost” linear vibrating mode. However, received in this way one mode response may not be correct if nonlinear terms play an essential role. The approach can be extended by making assumption that vibration modes of a forced system are close to the modes of free vibrations of a nonlinear conservative system. If we are able to find nonlinear normal modes of the nonlinear conservative system then on this basis we can find resonance response for “relatively” strong nonlinear models. To formulate nonlinear vibration modes we separate nonlinear conservative forces. It is worth to note that in general it is difficult to find a solution of the system (4), therefore the formal small parameter is still kept. We seek the approximate solutions in the form

\[ X = a u(a) \cos \omega_0 \tau \]  

where \( a \) and \( \omega_0 \) are amplitude and frequency of free vibrations and \( u(a) \) is an amplitude dependent eigenvector. For the convenience its coefficients are normalised with respect to the first coefficient, which is assumed to be equal one, \( u = \text{col}[1, u_2(a), \ldots, u_n(a)] \).

Substituting the solution (5) into (4), expanding all nonlinear terms in Fourier series and then grouping them in proper harmonics we can find approximate solutions by the harmonic balance method. In the first order approximation we neglect all terms of harmonics of order higher than one. This simplification leads to a set of nonlinear algebraic equations that relates modes coefficients with amplitude, \( u_j = f_j(a), j = 1, 2, \ldots, n \). Having such formulated amplitude dependent modal coefficients we can transform coordinates (Eq.(2))

\[ X = u(a)Y \]  

Substituting (6) to (4) and multiplying by \( u^T(a) \) we can notice that for a periodic function \( Y_j = a \cos \omega_0 \tau \), the system is reduced to a set of independent nonlinear differential equations

\[ M_j(a) \dot{Y}_j + M_j(a)\omega^2_{0j}(a) Y_j = 0, j = 1, 2, \ldots, n \]  

where \( M_j = m_1 u_1^2 + m_2 u_2^2 + \ldots + m_n u_n^2 \) are modal masses, and \( \omega_{0j} \) is frequency of the nonlinear system found from the first balanced harmonic. The details of formulation of Resonant Nonlinear Modes has been published in papers [5], [6].

4. Resonant Linear and Nonlinear Normal Modes of Viscous Damped Nonlinear Model

As an example we consider a two degree of freedom system presented in Fig.1 excited by external periodic force. The system may have relatively strong nonlinear elasticity and weak linear viscous damping.

![Figure 1 - Model of coupled nonlinear oscillators with viscous damping and external excitation](image-url)
Differential equations of motion are written as
\[
\begin{align*}
m_1 \ddot{X}_1 + c_1 \dot{X}_1 + \delta_1 X_1 + \gamma_1 X_1^3 + c_{12} \left( X_1 - X_2 \right) + \\
\delta_{12} \left( X_1 - X_2 \right) + \gamma_{12} \left( X_1 - X_2 \right)^3 &= q \cos \omega t \\
m_2 \ddot{X}_2 + c_2 \dot{X}_2 + \delta_2 X_2 + \gamma_2 X_2^3 - c_{12} \left( X_1 - X_2 \right) - \\
\delta_{12} \left( X_1 - X_2 \right) - \gamma_{12} \left( X_1 - X_2 \right)^3 &= 0
\end{align*}
\]
(8)

Taking into account transformation (2), the differential equation of motion (3) take form
\[
\begin{align*}
M_1 \ddot{Y}_1 + M_1 \omega_0^2 Y_1 &= \epsilon \left( u_{11} F_1 + u_{21} F_2 \right) \\
M_2 \ddot{Y}_2 + M_2 \omega_0^2 Y_2 &= \epsilon \left( u_{12} F_1 + u_{22} F_2 \right)
\end{align*}
\]
(9)

where
\[
\begin{align*}
M_1 &= m_1 u_{11}^2 + m_2 u_{21}^2, \\
M_2 &= m_1 u_{12}^2 + m_2 u_{22}^2, \\
F_1 &= \tilde{F}_1 + \tilde{F}_{12} + \tilde{F}_{d1} + \tilde{F}_{c1}, \\
F_2 &= \tilde{F}_{12} + \tilde{F}_{s12} + \tilde{F}_{d2} - \tilde{F}_{d12} + \tilde{F}_{c2}
\end{align*}
\]
and
\[
\begin{align*}
\tilde{F}_{1} &= -\tilde{\gamma}_1 \left( u_{11} Y_1 + u_{12} Y_2 \right)^3 \\
\tilde{F}_{2} &= -\tilde{\gamma}_2 \left( u_{21} Y_1 + u_{22} Y_2 \right)^3 \\
\tilde{F}_{s12} &= -\tilde{\gamma}_{12} \left[ \left( u_{11} + u_{12} \right) Y_1 - \left( u_{21} + u_{22} \right) Y_2 \right]^3 \\
\tilde{F}_{d1} &= -\tilde{\alpha}_1 \left( u_{11} Y_1 + u_{12} Y_2 \right) \\
\tilde{F}_{d2} &= -\tilde{\alpha}_2 \left( u_{21} Y_1 + u_{22} Y_2 \right) \\
\tilde{F}_{d12} &= -\tilde{\alpha}_{12} \left[ \left( u_{11} + u_{12} \right) Y_1 - \left( u_{21} + u_{22} \right) Y_2 \right]^3 \\
\tilde{F}_{c1} &= u_{11} \epsilon_1 \cos \omega t, \\
\tilde{F}_{c2} &= u_{12} \epsilon_1 \cos \omega t
\end{align*}
\]

Note that the frequencies are functions of vibration amplitude and the mode coefficients are normalised in such a way that \( u_{ij} = 1 \).

Applying nonlinear normal modes the differential equation of motion take form
\[
\begin{align*}
M_1 (a) \ddot{Y}_1 + C_1 (a) \dot{Y}_1 + M_1 (a) \omega_{01}^2 (a) Y_1 &= Q_1 \cos \omega t \\
M_2 (a) \ddot{Y}_2 + C_2 (a) \dot{Y}_2 + M_2 (a) \omega_{02}^2 (a) Y_2 &= Q_2 \cos \omega t
\end{align*}
\]
(12)

where modal masses \( M_1 (a), M_2 (a) \) are defined by Eq(7), and
\[
\begin{align*}
C_1 &= c_1 u_{11} + c_2 u_{21} + c_{12} \left( u_{11} - u_{21} \right)^2 \\
C_2 &= c_1 u_{12} + c_2 u_{22} + c_{12} \left( u_{12} - u_{22} \right)^2
\end{align*}
\]
(13)

are modal damping coefficients, while
\[
\begin{align*}
Q_1 &= q_1 u_{11}, \\
Q_2 &= q_1 u_{12}
\end{align*}
\]
(14)
denote modal amplitudes of excitation.

We may notice the difference in a structure of the equation (12) and equation (9). Damping in Eq.(12) is the term which has not been included in the nonlinear modes formulation, therefore should be assumed as small. The solution of the first and the second of Eq.(12) is obtained immediately in form
\[
\begin{align*}
Y_1 &= a_1 \cos \left( \omega t - \varphi_1 \right) \\
Y_2 &= a_2 \cos \left( \omega t - \varphi_2 \right)
\end{align*}
\]
(15)

where amplitude and phase are determined from equations known from theory of linear oscillators
\[
\begin{align*}
a_1 &= \frac{Q_1}{M_1 \sqrt{\left( \omega_{01}^2 - \omega^2 \right)^2 + \left( \frac{C_1 \omega}{M_1} \right)^2 \omega^2}}, \\
a_2 &= \frac{Q_2}{M_2 \sqrt{\left( \omega_{02}^2 - \omega^2 \right)^2 + \left( \frac{C_2 \omega}{M_2} \right)^2 \omega^2}}, \\
tan \varphi_1 &= \frac{C_1 \omega}{M_1 \left( \omega_{01}^2 - \omega^2 \right)}, \\
tan \varphi_2 &= \frac{C_2 \omega}{M_2 \left( \omega_{02}^2 - \omega^2 \right)}
\end{align*}
\]
(16)

We have to remember that Eq.(16) are to be solved together with constrain equation (10) and the frequency equation (11) as functions of amplitude.

Exemplary analysis is performed for the following numerical data
\[
\begin{align*}
m_1 &= 1, m_2 = 2, \delta_1 = 1, \delta_2 = 1, \delta_{12} = 0.3, \\
\gamma_1 &= 0.1, \gamma_2 = 0.1, \gamma_{12} = 0
\end{align*}
\]
(17)

Linear vibration modes are represented by vectors, mode I: \( u_{120} = \text{col}[1, 2.37702] \), mode II: \( u_{120} = \text{col}[1, -0.21035] \). Index zero points out the linear coefficients. Natural linear frequencies are: \( \omega_{01} = 0.7661, \omega_{02} = 1.1675 \).

The resonance curves determined by linear (LNM) and nonlinear (NNM) normal modes are presented in Fig.2. As can be seen the resonance region found by various modes formulation leads to essentially different results. Near the first natural frequency \( \omega_{01} \) the resonance zone and the curve determined by NNM are much smaller then those received by classical one mode approach. Opposite situation takes place near the second natural frequency \( \omega_{02} \), where linear approach (LNM) reduces real resonance region and its amplitudes.
5. NONLINEAR NORMAL MODES OF FORCED SELF-EXCITED OSCILLATORS

In this chapter we apply the formulated NNM for a more complex system composed of coupled oscillators presented in Fig. (1), however damping of the oscillators is assumed to be nonlinear and producing self-excitation, moreover apart from external excitation, parametric coupling is assumed due to periodically changing stiffness of the coupling spring. Differential equations of motion for such a system take form

\[
m_1 \ddot{X}_1 + f_{d1} \left( \dot{X}_1 \right) + \delta_1 X_1 + \gamma_1 X_1^3 + \left( \delta_1 - \mu \cos \vartheta \tau \right) \times (X_1 - X_2) = q \cos \omega \tau
\]

\[
m_2 \ddot{X}_2 + f_{d2} \left( \dot{X}_2 \right) + \delta_2 X_2 + \gamma_2 X_2^3 - \left( \delta_2 - \mu \cos \vartheta \tau \right) \times (X_1 - X_2) = 0
\]

(18)

Let us assume that damping is strongly nonlinear represented by Rayleigh’s terms, \( f_{d1} = -\alpha_1 \dot{X}_1 + \beta X_1^2 \), \( f_{d2} = -\alpha_2 \dot{X}_2 + \beta X_2^2 \). Resonance normal modes formulated in this paper can be used properly under the condition that the response of the system is periodic. The periodic response of the model (18) is observed near the resonance regions. In these regions, the self-excitation frequency is quenched by excitation. Therefore, in spite of existence of the nonlinear self-excited terms motion of the system is periodic. Over those regions the response is quasi-periodic and then another approach is desired. The nonlinear modes are determined near resonance regions making the same assumption as before that a mode shape of a nonlinear conservative system is close to that of a full complex structure. Applying the procedure described in chapters 3 and 4 we received reduced model of the considered problem

\[
M_j \ddot{Y}_j + M_j \omega_{0j}^2 Y_j - C_{\alpha j} \dot{Y}_j + C_{\beta j} Y_j^3 - C_{\mu j} Y_j \cos 2\theta \tau = Q_j \cos \omega \tau
\]

(19)

Modal mass \( M_j \), natural frequency \( \omega_{0j} \), and external excitation \( Q_j \) remain with the same definitions as in the former chapter. New parameters occurring in (19) correspond to self- and parametric excitation and they are defined as \( C_{\alpha j} = \alpha_1 u_{1j}^2 + \alpha_2 u_{2j}^2 \), \( C_{\beta j} = \beta_1 u_{1j}^4 + \beta_2 u_{2j}^4 \) - coefficients of nonlinear modal damping, \( C_{\mu j} = (u_{1j} - u_{2j})^2 \) - modal amplitude of parametric excitation and frequency \( 2\theta \). The new terms complicate the solutions comparing with the former example. To get solution an approximate method has to be used. Therefore, equation (19) is written in form with a formal small parameter engaged in the procedure

\[
M_j \ddot{Y}_j + M_j \omega_{0j}^2 Y_j = \varepsilon \left( C_{\alpha j} \dot{Y}_j - C_{\beta j} Y_j^3 \right) + \tilde{C}_{\mu j} Y_j \cos 2\theta \tau + \tilde{C}_{\mu j} \cos \omega \tau \]

(20)

where \( C_{\alpha j} = \varepsilon \tilde{C}_{\alpha j} \), \( C_{\beta j} = \varepsilon \tilde{C}_{\beta j} \), \( C_{\mu j} = \varepsilon \tilde{C}_{\mu j} \), \( Q_j = \varepsilon \tilde{C}_{\mu j} \). We assume small and positive value of \( \varepsilon \). According to multiple scale method we seek a periodic solution in series of a small parameter

\[
Y_j(\tau, \varepsilon) = Y_{j0}(T_0, T_1) + \varepsilon Y_{j1}(T_0, T_1) + \ldots
\]

(21)

near the resonance region

\[
\vartheta^2 = \omega_{0j}^2 + \varepsilon \sigma_j
\]

(22)

where \( \sigma_j \) is a detuning parameter near the first or the second resonance region. Substituting solution (21) into differential equations of motion (20) we get set of recurrent equations in successive perturbation orders

\[
D_{y0} Y_{j0} + \vartheta^2 Y_{j0} = 0
\]

\[
D_{y1} Y_{j1} + \vartheta^2 Y_{j1} = \left[ \sigma_j Y_{j0} - 2D_{y0} D_{y1} Y_{j0} + \tilde{C}_{\alpha j} D_{y0} D_{y1} Y_{j0} - \tilde{C}_{\beta j} (D_{y1} D_{y0})^3 + C_{\mu j} Y_{j0} \cos 2\theta \tau + \tilde{C}_{\mu j} \cos \omega \tau \right] / M_j
\]

(23)

Further analysis is carried out for two variants, (a) a case of parametric and self-excited system, \( \tilde{C}_{\mu j} = 0 \), and (b) a case of self-excited system with parametric and external excitation, \( \tilde{C}_{\mu j} > 0 \) and \( \omega = \vartheta \). A secular terms elimination
The approximate solution has form

\[ \dot{\theta} = C_{\varphi j} \sin \varphi \]

Equation (25) has to be solved together with constraint equation (10). The approximate solution has form

\[ C_{\varphi j} = \sqrt{\lambda_j + C_{\varphi j} \sqrt{2C_{\varphi j}^2 + 4a_j^2C_{\mu j}\lambda_{2j} - 2\sqrt{\lambda_j}}} = 0 \]

where

\[ \lambda_{1j} = a_j (C_{\alpha j}\omega - \frac{3}{2}a_j^2C_{\beta j}\omega^3), \quad \lambda_{2j} = M_j (\omega_{0j}^2 - \omega^2), \quad \Lambda_j = (C_{q j} - 2a_j^2C_{\mu j}\lambda_{2j})^2 + 4a_j^2C_{\mu j}(C_{\mu j}\lambda_{2j}^2 + 2C_{\varphi j}^2\lambda_{2j}) \]

Equation (25) has to be solved together with constraint equation (10). The approximate solution has form

\[ Y_j = a_j \cos (\omega t + \varphi) + \varepsilon \frac{a_j^3}{16M_j} \left[ -\tilde{C}_{\mu j} \cos (3\omega t + \varphi) + \frac{a_j^2}{4} \tilde{C}_{\beta j} \omega \sin (3\omega t + 3\varphi) \right] \]

If the coefficient \( C_{\varphi j} \) is taken as equal to zero, then we get resonance caused by parametric excitation, otherwise both, parametric and externally forced vibration occur. It is worth to note that the formulated nonlinear normal modes are valid for periodic system’s response. Modulation equations (24) allows to determine solution also for non stationary regimes which is in contrast to assumption made in nonlinear mode formulation. However, we may expect that for slowly varying amplitude and phase the formulation should remain valid.

Numerical example is presented for data

\[ m_1 = 1, m_2 = 2, \alpha_1 = 0.01, \beta_1 = 0.05, \]
\[ \alpha_2 = 0.01, \beta_2 = 0.05, \delta_{12} = 0.3, \mu = 0.3, q = 0, \text{ or, } \mu = 0.2, q = 0.1 \]

The modal coefficients take values

- for first mode; \( M_1 = 12.3004, C_{\mu 1} = 0.3792, C_{\alpha 1} = 0.0665, C_{\beta 1} = 1.6462 \)
- for second mode; \( M_2 = 1.0885, C_{\mu 2} = 0.2930, C_{\alpha 2} = 0.01044, C_{\beta 2} = 0.0501 \)

The resonance curves calculated for the assumed data near the first natural frequency \( \omega_{01} \) are presented in Fig.3. The surface shows resonance region versus parametric excitation amplitude \( \mu \) and frequency \( \vartheta \) calculated by the linear normal mode transformation. For comparison a cross-section for \( \mu = 0.3 \) is taken and presented in Fig.4(a). Amplitudes obtained by nonlinear modes (NNM) are essentially smaller than those got by classical linear modes (LNM). Also the NNM resonance zones fit much better to direct numerical simulations [6]. The mode coefficient \( u_21 \) versus amplitude is plotted in Fig.4(b). For a linear system this coefficient is independent of amplitude (the horizontal line).

Periodic resonances looks totally different if external force additionally excites the system.

An exemplary bifurcation diagram amplitude of \( a \) versus external excitation \( q \) obtained for a one DOF model is presented in Fig.5. It is clearly visible that for selected parameters there are possible five nontrivial solutions [1], [2]. For amplitude \( q = 0 \) we get two nontrivial and one trivial solutions (starting points in vertical axis) and then increasing \( q \) we get five branches of nontrivial solutions which transit into three and eventually to one solution. For the considered reduced order model the resonance curve should exhibit five nontrivial solutions in the resonance zones. Numerical
Figure 5 – Bifurcation diagram vibration amplitude versus amplitude of excited force \( q \) and fixed excitation frequency \( \vartheta = 1.07; \alpha = 0.01, \beta = 0.05, \gamma = 0.1, \mu = 0.2 \)

Figure 6 – Resonance curves around \( \omega_{02} \); nonlinear modes formulation (a), linear normal modes (b)

simulations shows that in fact for selected set of parameters there are multiple solutions in the resonance zones [12]. Taking into account this observation the set of nonlinear equations (25) and (10) has to be solved together. The solution has been received by solving numerically nonlinear algebraic equations for various starting points and then a continuation method used to get resonance branches. The resonance curve determined by NNMs near the second natural frequency are presented in Fig.6(a). The additional solutions are represented by a loop in the resonance curve and there are five possible nontrivial solutions in this regions. Stability analysis shows that only two upper are stable. This result is in a good agreement with numerical simulations (see [12] for details). Linear normal modes transformation leads to a resonance curve presented in Fig.6(b). We see an important difference between both results. The loop is much bigger and is located in a different place, on the left side of the resonance. Moreover, only three nontrivial solutions occur, instead of expected five. It is clearly seen that the proposed NNMs essentially improve quality of the reduced model.

6. CONCLUSION

The paper presents a formulation of resonant nonlinear normal modes which allows to reduce dynamics of a many degrees of freedom system into independent decoupled nonlinear oscillators. The reduction is possible if the response of the system is periodic. Therefore, the method can be applied mainly for the resonance region. The formulated nonlinear modes differs from that presented e.g. in [7], [9]. The modes are not time dependent but related with the system’s parameters and vibration amplitude. The mode coefficients are determined as functions of amplitude which leads to an additional constrain equation. In this way the resonance curve has been corrected with regards to classical linear transformation. The modes can be successfully applied for a system with weak damping or a system in which damping is eliminated by system’s dynamics [10], [11]. The effectiveness of the method is tested for nonlinear oscillator with weak viscous damping and forced by a harmonic force, and then extended for self-excited oscillators driven parametrically and externally. The presented examples show important differences in resonant response for linear and nonlinear modes approach. The results received by formulated nonlinear normal modes fit very well to direct numerical simulations results.

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