H2 OPTIMAL CONTROL FOR SMART TRUSS STRUCTURE

Gustavo Luiz C. M. de Abreu 1, Vicente Lopes Jr. 2

1 UNESP de Ilha Solteira, Departamento de Engenharia Mecânica, Ilha Solteira-SP, Brasil, gustavo@dem.feis.unesp.br
2 UNESP de Ilha Solteira, Departamento de Engenharia Mecânica, Ilha Solteira-SP, Brasil, vicente@dem.feis.unesp.br

Abstract: This paper presents an active damping control of a smart truss structure using pair of piezoelectric stack actuators collinear with force transducers and a H2 optimal control strategy. A finite element model of the structure is constructed using the three-dimensional frame elements subjected to axial, bending and torsional loads considering the electro-mechanical coupling between the host structure and piezoelectric stack actuators. In this paper, the active member placement is determined by using the fraction of modal strain energy as an optimal index. The performance objectives and the high frequency unmodeled dynamics are considered in the controller design to produce a H2 optimal controller based on force feedback. The purpose of the controller is to minimize the effect of disturbances on the entire structure. The numerical simulation results demonstrate that the active piezoelectric strut actuator and the H2 control strategy can effectively reduce truss vibration in the presence of several operating conditions.

Keywords: H2 Optimal Control, Smart Truss Structure, Piezoceramic Stack Actuators.

1. INTRODUCTION

A truss structure is one of the most commonly used structures in aerospace and civil engineering [1]. Because it is desirable to use the minimum amount of material for construction, the trusses are becoming lighter and more flexible which means they are more susceptible to vibration. A convenient way of controlling a truss structure is to incorporate a piezoelectric stack actuator into one of the truss members [2].

Research on damping of truss structures began in the late 80’s. Fanson [3], Chen et al. [4] and Anderson et al. [5] developed active members made of piezoelectric transducers. Preumont et al. [6] used a local control strategy to suppress the low frequency vibrations of a truss structure using piezoelectric actuators. Their strategy involved the application of integrated force feedback using two force gauges each collocated with the piezoelectric actuators, which were fitted into different beam elements in the structure. Carvalhal et al. [7] used an efficient modal control strategy for the active vibration control of a truss structure. In that approach, a feedback force is applied to each node to be controlled according to a weighting factor that is determined by assessing how much each mode is excited by the primary source.

The choice of the actuator/sensor location is an important issue in the design of actively controlled structures. The actuators/sensors should be placed at locations so that the desired modes are excited most effectively [8]. A wide variety of optimization algorithms were proposed to this end in the literature. Two popular examples are the Simulated Annealing method [9] and the Genetic Algorithm method [10]. In [11] a review of the different placement strategies for sensor and actuator was presented. Although these methods are effective, they fail to give a clear physical justification for the choice of the actuator/sensor placement. In this paper, it was chosen a more physical method that has been used by [6]. It merely consists in placing the transducer in the truss structure with maximal fraction of modal strain energy which is a measure that represents the ability of a vibration mode to concentrate the vibrational energy into the actuator and can be interpreted as a compound indicator of controllability and observability of the specific mode by the actuator.

In control design problems, an interest in designing a controller for a particular frequency range is often manifested. In many structures the lower modes of vibration have the most energy and are hence more critical. For this reason, these modes are often targeted for active control so that energy is not wasted in controlling the higher modes of vibration [12]. In such situations, the rest of the dynamics of the structure in the high frequency region can cause problems if not included in the controller design. These unmodeled dynamics are known to cause spillover problems, which occur due to excitation of the uncontrolled modes leading to degradation in performance and even instability. This uncertainty is modeled by means of a weighting function that serves as a weighting factor to balance the controller effort with respect to the degree of vibration reduction that can be achieved. In practice, a suitable weighting function can be determined to find a controller with sufficient damping properties at lower frequency range and robustness to higher-order unmodeled dynamics.

This paper describes a standard H2 optimal controller design framework for suppressing the undesired structural vibrations in a truss structure containing piezoelectric actuators and collocated force sensors forming a so-called smart/intelligent truss structure.
2. THE TRUSS STRUCTURE

The truss structure used in this work is depicted in Fig. 1. It consists of 12 bays of 140 mm each, made of steel bars of 4 mm diameter connected with plastic joints (mass block of 70g) and clamped at the bottom. It is equipped with active member as indicated in the Fig. 1. It consists of a piezoelectric linear actuator collinear with a force transducer.

![Fig. 1. Truss structure with an active member](image)

2.1. Governing Equations

Consider the linear structure of Fig. 1 equipped with a discrete, massless piezoelectric stack transducer. The equation governing the motion of the structure excited by a force \( f \) and controlled by the piezoelectric actuator \( f_a \) is

\[
M\ddot{x} + C\dot{x} + Kx = bf + b_a f_a
\]

(1)

where \( K \) and \( M \) are the stiffness and mass matrices of the structure, obtained by means of the finite element model using the three-dimensional frame elements [13] (each node has six degrees of freedom), \( C \) is the damping matrix; \( b \) and \( b_a \) are, respectively, the influence vectors showing the locations of the external forces \( f \) and the active member in the global coordinates of the truss (the non-zero components of \( b_a \) are the direction cosines of the active bar in the structure), and \( f_a \) is the force exerted by the active member.

Consider the piezoelectric linear transducer of Fig. 1 is made of \( n_a \) identical slices of piezoceramic material stacked together. The force exerted by the active member is defined by [14]

\[
f_a = -K_a (\Delta - n_a d_{33} V)
\]

(2)

where \( K_a \) is the stiffness of the actuator, \( d_{33} \) is the piezoelectric coefficient, \( V \) is the voltage applied to the piezo that produces an free expansion \( \delta \), and \( \Delta \) is the projection of the displacements at the end nodes of the active member i.e., \( \Delta \) is the sum of the free piezoelectric expansion (\( \delta \)) and the elastic displacement (\( f_a / K_a \)).

The elongation \( \Delta \) is linked to the structural displacement by

\[
\Delta = b_a^T x
\]

(3)

The equation governing the structure containing the active member can be found by substituting Eqs. (2) and (3) with Eq. (1); the new equation is

\[
M\ddot{x} + C\dot{x} + \left( K + K_a b_a b_a^T \right) x = bf + b_a K_a n_a d_{33} V
\]

(4)

where \( K \) is the stiffness matrix of the structure excluding the axial stiffness of the actuator.

The equation (4) can be transformed into modal coordinates according to \( x = \Phi \eta \), where \( \Phi = [\phi_1 \ \phi_2 \ \ldots \ \phi_n] \) is the matrix of the mode shapes, solutions of the eigenvalue problem: \( M\Phi\Phi^T = \Phi \lambda \Phi^T \). Assuming normal modes normalized according to \( \Phi^T M\Phi = I \) and introducing the modal state vector \( x_i = [\eta_i \ \eta_i] \), the transformed equation of motion (4) becomes

\[
\dot{x}_i = A_{ii} x_i + B_{ii} f + B_{i2} V
\]

(5)

where

\[
A_{ii} = \left[ \begin{array}{cc} 0 & I \\ -\bar{K} & -\bar{C} \end{array} \right], \quad B_{ii} = \left[ \begin{array}{c} 0 \\ \Phi^T b_a K_a n_a d_{33} \end{array} \right],
\]

and \( \bar{K} = \text{diag}\{\omega_i^2\}, \bar{C} = \text{diag}\{2\zeta_i \omega_i\} \), \( \omega_i \) is the \( i \)-th natural frequency of the truss and \( \zeta_i \) is the associated modal damping.

Similarly to Eq. (2), the output signal of the force sensor, proportional to the elastic extension of the truss, is defined by

\[
y = C_{i2} x_i + D_{i22} V
\]

(7)

where

\[
C_{i2} = \left[ -K_a b_a^T \Phi \right], \quad D_{i22} = K_a n_a d_{33}
\]

(8)

2.2. Actuator Placement

More than any specific control law, the location of the active member is the most important factor affecting the performance of the control system. Good control performance requires the proper location of the actuator to achieve good controllability. The active member should be placed where its authority over the modes it is intended to control is largest. It can be achieved if one locates the transducer in order to maximize the mechanical energy stored in it. The ability of a vibration mode to concentrate the vibrational energy into the transducer is measured by the fraction of modal strain energy \( y_i \) defined by [6]
The Eq. (9) is readily interpreted as the ratio between the strain energy in the actuator and the total strain energy when the structure vibrates according to $i$-th mode. Physically, $v_i$ can be interpreted as a compound indicator of controllability and observability of mode $i$ by the transducer. The best location consists in placing the transducer in the truss structure with maximal fraction of modal strain energy $v_i$, where $i$ is the mode to be controlled.

In this work, the control objective is to damp the first two modes of the structure by using two active elements. The search for candidate locations where these active members can be placed is greatly assisted by the examination of the first two structural mode shapes as can be seen in Fig. 2.

From the Fig. 2 and the Tab. 1 one can see that active member number 2 has a large influence on mode 1 and almost no influence on mode 2, and that the opposite occurs for active member number 3 i.e., substituting the active member for the bar 2 provides a strong control on mode 1 ($v_1 = 4.27\%$), but offers a reasonable control on mode 2 ($v_2 = 0.92\%$). By contrast, an active member substituted for the bar 3 offers almost no control on mode 1 ($v_1 = 0.23\%$) and good controllability on mode 2 ($v_2 = 5.44\%$). This result motivated the positions of the transducers in the actual truss.

3. $H_2$ OPTIMAL CONTROLLER DESIGN

Consider the general block diagram description of the control problem given in Fig. 3. In this figure, $P$ and $K$ represent, respectively, the generalized plant and the controller transfer functions, $y$ is the output vector of measured forces, $z$ is the controlled output, $u$ is the control voltage signals, and $d$ is the input vector of excitations representing external disturbance forces $f$ and measurement noise $n$.

![Fig. 2. a) disposition of the active elements; b) first mode shape and c) second mode shape](image)

![Fig. 3. Block diagram of general control system with output feedback.](image)

Considering the main characteristics of both transducers as: $K_u = 65\ N/\mu m$ and $n_d d_3 = 6.315\times10^{-7} m/Volts$, the fractions of modal strain energy $v_i$ computed from Eq. (9), are shown in Table 1 for the first six structural elements (see Fig. 2a).

<table>
<thead>
<tr>
<th>Element</th>
<th>$v_1$ (%)</th>
<th>$v_2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.92</td>
<td>2.60</td>
</tr>
<tr>
<td>2</td>
<td>4.27</td>
<td>0.92</td>
</tr>
<tr>
<td>3</td>
<td>0.23</td>
<td>5.44</td>
</tr>
<tr>
<td>4</td>
<td>1.46</td>
<td>3.23</td>
</tr>
<tr>
<td>5</td>
<td>3.36</td>
<td>0.50</td>
</tr>
<tr>
<td>6</td>
<td>0.21</td>
<td>4.05</td>
</tr>
</tbody>
</table>

The plant $P$ contains the structural system plus filters and weighting functions in the frequency domain as shown in Fig. 4. The regulated output vector $z$ may consist of any combination of the states of the system and components of the control input vector $u$ weighted by the frequency dependent function $W_u$. 

![Proceedings of the 9th Brazilian Conference on Dynamics Control and their Applications](image)
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The task here is to design the controller $K$ such that it stabilizes the system and, within the class of all controllers $H_2$ controllable and ($C_2$, $A$) detectable. Hence, the full order $H_2$ controller is expressed by

$$
\begin{bmatrix}
\dot{x}_c \\
u
\end{bmatrix} = \begin{bmatrix}
A_k & B_k
\end{bmatrix} \begin{bmatrix}
x_c \\
y
\end{bmatrix};
K = \begin{bmatrix}
A_k & B_k \\
C_k & 0
\end{bmatrix}
$$

where

$$
A_k = A + K_f C_2 + B_2 K_c + K_f D_{22} K_c
$$

and

$$
B_k = -K_f
$$

$$
C_k = K_c
$$

The matrices and $X$ and $Y$ are solutions to the corresponding algebraic Riccati equations

$$
\begin{pmatrix}
A - B_2 D_{21}^T C_1 \\
A - B_1 D_{11}^T C_1
\end{pmatrix} X + X \begin{pmatrix}
A - B_1 D_{11}^T C_1 \\
A - B_2 D_{21}^T C_1
\end{pmatrix} - X B_2 B_2^T X + C_1^T C_1 = 0 \\
y (A^T - C_2^T D_{21} B_2^T) + (A^T - C_1^T D_{11} B_1^T) y - Y C_2^T C_1 y + B_1 B_1^T = 0
$$

4. SIMULATIONS AND NUMERICAL RESULTS

Numerical simulations are presented to demonstrate the efficacy of the controller applied to the truss structure. The structure considered is the 12-bay of 140 mm each with 111 members and 41 nodes, and the nodes at the bottom are clamped (see Fig. 1). The passive members are made of steel with a diameter of 4 mm, and the damping is assumed to be proportional to the stiffness and mass matrices so that $C = 10^3 M + 10^2 K$. At each node there is a centralized mass block of 70g and has six degrees of freedom (dof), translations and rotations in $x$, $y$ and $z$ directions, so the truss structure has 228 active dofs, and the state-space model consequently has an order of 456. The strategy is to control the first two modes (17.80 Hz and 20.61 Hz) by using two active members positioned in the finite elements shown in Fig. 2a.

4.1. Controller Design

Assuming that the disturbance force are located on each node at the top of the truss and in all directions (see Fig. 1), in this paper, the purpose of the controller is to reduce the effect of external disturbance forces $d$ on the entire structure. The regulated output vector $z$ is then selected to be the displacements of all nodes and in three translational directions ($x$, $y$, and $z$). The resultant global location matrix of the regulated outputs and of the disturbance forces must be transformed into modal coordinates to obtain the corresponding modal matrices $B_{11}$ (Eq. 6) and $C_{11}$ (see Fig. 4).

Because the controller is designed to damp the lower modes, it is important to ensure the robustness of the closed-loop system to unmodeled dynamics that can cause spillover problem and occur due to excitation of the uncontrolled modes. This problem can be fixed by introducing a weight on the control signals $u$. The weighted function $W_u$ is then determined to find a controller with sufficient damping properties in the lower frequency range and robustness to higher-order unmodeled dynamics. The basic assumption to construct $W_u$ is that the control energy should not be wasted in controlling the higher modes of vibration. Therefore, the weighted function $W_u$ is selected as

$$
\|T_{sd}\|_2 = \text{trace} \left[ \frac{1}{2\pi} \int_{-\infty}^{\infty} T_{sd}^*(j\omega) T_{sd}(j\omega) d\omega \right]
$$
and the frequency response is plotted in Fig. 6, where $V_{\text{max}}$ is the maximum control voltage required (150 Volts), $\omega_n$ is the second natural frequency (130 rad/s) and $\zeta$ is equal to 0.5.

As illustrated in Fig. 5, the control weighting function $W_u$ allows to limit the control voltage in the frequency range containing the first two structural vibration modes ($\omega \leq 20$ Hz) and to reduce the magnitude of the control actions over the range of higher frequencies.

Based on the design approach of section 3, a 12-state $H_2$ optimal controller is synthesized.

### 4.2. Simulation Results

To verify the controller performance numerically, open loop and closed loop simulations were conducted and the results are presented and discussed. Impulsive forces are applied in all directions on each node at the top of the structure (see Fig. 1). The uncontrolled and controlled responses of the force transducers 1 and 2 in time domain are shown in Figs. 6 and 7. This type of force is used as it will excite many modes of vibration and hence is a difficult test for the control system. From the results it can be observed that the sensor responses are reduced.

A time-varying chirp forces $f$ (amplitude of 50 N) from 1 Hz to 20 Hz with a target time of 3 seconds are applied in all directions on each node at the top of the structure (see Fig. 1). The uncontrolled and controlled responses of the force transducers 1 and 2 in time domain are shown in Figs. 8 and 9. Figure 10 presents the corresponding control voltages.
5. CONCLUSIONS

A $H_2$ optimal controller was designed and numerically implemented on a truss structure containing a pair of piezoelectric linear actuators collinear with force transducers. The procedure used for placing actuators along the structure was proven to be effective. Besides, the optimal index has a strong intuitive appeal. The optimal controller, obtained by solving a standard $H_2$ control problem, was designed to reduce the effect of external disturbance forces on the entire structure. A set of numerical simulations was performed, which demonstrated the effectiveness of the developed controller in reducing the vibrations of a truss structure. The inclusion of uncertainties in the controller design and the experimental tests however, are relegated to the future.

ACKNOWLEDGMENTS

The first author would like to thank the FAPESP (Nº 2008/05129-3) for the financial support of the reported research.

REFERENCES


