SEMI-ACTIVE SUSPENSION CONTROL WITH ONE MEASUREMENT SENSOR USING $H_\infty$ TECHNIQUE

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Abstract: In this paper the $H_\infty$ control strategy is addressed. This control technique was tested in a system where saturation is present, the semi-active suspension problem and was evaluated using performance indexes regarding comfort and road handling, and solved by the use of some specific tools of the MATLAB software. The main goal is to obtain optimal performance, in order to minimize the sprung mass (chassis) acceleration and to ensure road-holding characteristics. A comparison is then made with a passive system, by means of numerical simulation, using a quarter-car model, real road profiles, by means of the performance indexes and frequency response comparisons. A single-sensor approach is also suggested, to reduce the number of sensors used per quarter-car model.

Keywords: semi-active suspension, h-infinity control, automotive control

1. INTRODUCTION

The vehicle suspensions are used to isolate the passenger compartment from any road irregularities, while supporting the vehicle, counteracting internal and external forces. Initially, the need for suspensions was only related to passenger comfort, but nowadays there are also concerns regarding vehicle handling.

Suspension systems are divided in either passive or active systems, and the latter can be further divided into other two categories, fully active systems and semi-active systems.

Passive suspension systems are built with static springs and dampers. This means that their parameters must be chosen adequately to achieve comfort and road handling, while under different running conditions. So a balance between comfort and safety must be chosen, leading to a trade-off, where improving one parameter leads to degrading the other. This trade-off between safety and comfort has motivated the study of active suspensions.

The fully active suspension systems are usually composed of pneumatic or hydraulic actuators, which can introduce, store or dissipate energy into and from the system. As a result, the conflict between comfort and road handling is better resolved. On the other hand, these actuators have a high cost and weight, and require a reasonable amount of energy to be used, greatly increasing the system’s energy consumption.

Semi-active suspension systems make use of dampers with varying damping coefficient, such as a hydraulic damper with variable orifice size. These devices are very efficient, as they require negligible additional power and consequently, provide a good compromise between cost and performance ([5, 17]).

The goal of this paper is to suggest a semi-active control technique, based on $H_\infty$ control, and compare it to a typical passive suspension.

One main difficulty of implementing semi-active control systems is related to measuring system states, as well as sensor costs. This study proposes the use of a single sensor per quarter of the vehicle, as a means of reducing costs and working around unmeasurable states.

1.1. Quarter-car model

The quarter-car system is widely used as the system to control, in the suspension control scenario (see [9], [7] and [16]). A quarter-car schematics is shown in Figure 1, where the main chassis is represented by a rigid mass, and the wheel is represented as a rigid mass plus spring.

The model parameters are:

- $m_c$: chassis mass;
- $m_w$: wheel mass;
- $k_w$: tire stiffness;
• $k$: spring stiffness;

And the model variables are:

• $z_r$: road profile;
• $z_w$: wheel vertical position;
• $z_c$: chassis vertical position;
• $i$: control current of semi-active damper;
• $\beta_c$: variable damping coefficient.

1.2. The semi-active damper

In the semi-active suspension system, the actuator is a damper with variable characteristics. In this paper, a model of a semi-active damper that uses a variable orifice is used. So the actuator limitations come from the inability to add power to the system, as a regular hydraulic actuator could.

The actuator force in the damper is also a function of the relative velocity between its extremities, so the control force is determined as $u(v_{w,c}) = \beta(z_w(t) - \dot{z}_c(t))$, where $\beta$ is variable. Using this information, a damper map can be drawn. This mapping relates the feasible actuator force to the relative velocity between the damper, and is shown on Figure 2. The gray area represents the whole range of feasible forces and the lines represent the orifice’s maximum and minimum openings.

The controller must supply the necessary current to open the valve at each instant. To be able to calculate that, it is necessary to also measure the relative velocity $v_{w,c}$ at each instant. So the dynamic equations of the quarter-car model are given in equation (1).

$$\begin{align*}
    m_c \ddot{z}_c &= -k(z_c - z_w) + u \\
    m_w \ddot{z}_w &= k(z_c - z_w) - u - k_w(z_w - z_r)
\end{align*}$$

(1)

The quarter-car can be better represented as a state space system, for the purpose of designing the control system. For this paper, the state variables were chosen as:

$$x_1 = z_c \quad x_2 = \dot{z}_c \quad x_3 = z_w \quad x_4 = \dot{z}_w$$

(2)

The state vector is then represented as $x = [x_1 \ x_2 \ x_3 \ x_4]$, where $x_1$ denotes the chassis absolute position, $x_2$ denotes the chassis velocity, $x_3$ represents the wheel absolute position and $x_4$ is the wheel velocity. There are also exogenous inputs to the system - the control input $u$, given by the semi-active damper and the road input $z_r$, that will be represented as the disturbance $d$ so that:

$$u = \beta(i) \cdot (\dot{z}_w(t) - \dot{z}_c(t)) \quad d = z_r$$

(3)

By this way, the system on (1) can be written using (2) and (3) as:

$$\dot{x} = Ax + B_1 d + B_2 u$$

(4)

and $A$, $B_1$ and $B_2$ are represented in (5):

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_{w} & 0 & k_{w} & 0 \\ 0 & 0 & 0 & 1 \\ k_{w} & 0 & -k_{w} & 0 \end{bmatrix}$$

(5)

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_{w}}{m_w} \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{1}{m_w} \end{bmatrix}$$

2. THE SEMI-ACTIVE CONTROL

2.1. Clipped control

The clipped control is a technique widely used in semi-active suspension control ([1, 3–5, 15]). It is basically a two-step procedure - first, a controller is designed assuming an active actuator system is present and then, at each instant, the control force is saturated according
to the damper’s limitations. The concept of the clipped control can be defined by equation (6).

\[
\begin{cases}
\text{if } v_{uc} > 0 \text{ and } u^* > u_{i,\text{max}}, \text{ then } u = u_{i,\text{max}}; \\
\text{if } v_{uc} > 0 \text{ and } u_{i,\text{min}} < u^* < u_{i,\text{max}}, \text{ then } u = u^*; \\
\text{if } v_{uc} > 0 \text{ and } u^* < u_{i,\text{min}}, \text{ then } u = u_{i,\text{min}}; \\
\text{if } v_{uc} < 0 \text{ and } u^* < u_{i,\text{min}}, \text{ then } u = u_{i,\text{min}}; \\
\text{if } v_{uc} < 0 \text{ and } u^* > u_{i,\text{min}}, \text{ then } u = u_{i,\text{min}}; \\
\text{if } v_{uc} < 0 \text{ and } u^* < u_{i,\text{max}}, \text{ then } u = u_{i,\text{max}}.
\end{cases}
\]  

(6)

Basically, the concept of the clipped strategy is to use a control strategy and to saturate it according to the actuator constraints.

2.2. $\mathcal{H}_\infty$ control

The $\mathcal{H}_\infty$ consists in minimizing the system’s $\mathcal{H}_\infty$ norm. Given a linear system, $G(s)$ as in equation (7):

\[
\begin{align*}
\dot{x} &= Ax + B_1w + B_2u \\
z &= C_1x + D_{11}w + D_{12}u \\
y &= C_2x + D_{21}w + D_{22}u
\end{align*}
\]  

(7)

where $x$ is the state vector, $z$ is the performance vector, $y$ denotes the measured variables, and $u$ is the control input vector. The $\mathcal{H}_\infty$ norm of $G(s)$ is such that:

\[
\|G\|_{\mathcal{H}_\infty} = \max_\omega |G(j\omega)|
\]  

(8)

Basically, the $\mathcal{H}_\infty$ norm represents the peak values of $G(s)$ in the frequency domain. The standard closed loop control configuration used for the control design can be seen in Figure 3.

In this figure, $w$ is the vector of exogenous inputs, $z$ is the performance vector, $u$ is the control input, and the vector $y$ consists of the measured variables.

The control objective consists of finding $K(s)$, given a system $G(s)$, so that the closed loop system is stable and that the system’s $\mathcal{H}_\infty$ norm is minimized. This is done by designing the sub-optimal controller that guarantees that the $\mathcal{H}_\infty$ norm of the closed-loop transfer function from $w$ to $z$ ($T_{zw}$) is lesser than $\gamma$ (9). Then, $\gamma$ is minimized through an iterative process.

\[
\|T_{zw}\| \leq \gamma
\]  

(9)

The clipped strategy, introduced in section 2, will be used in conjunction with the $\mathcal{H}_\infty$ control for the vehicle semi-active suspension system, increasing comfort and road handling characteristics.

2.3. State Estimator

The $\mathcal{H}_\infty$ together with the clipped strategy, requires many measurements, such as as the relative velocity $v_{uc}$, as well as the states required by the control input to be calculated. This leads to a higher implementation cost as well as a difficulty in measuring all the needed variables. To go around this problem, observers are designed and used ([1, 8, 11]). For this work, the widely used Kalman filter ([6, 13]) will be used.

Considering a linear, discrete dynamic system, described in (10):

\[
x_{k+1} = Ax_k + Bu_k + Gw_k \\
z_k = Hx_k + v_k
\]  

(10)

where $x_k$ is the state vector, $u_k$ the inputs, $z_k$ the outputs, $w_k$ is the process noise and $v_k$ is the measurement noise.

It is considered here that the process noise $w_k$ is random noise with mean $\bar{w}_k = 0$ and the covariance $\text{cov}(w_k, w_k) = w \sim (0, Q) = Q$, and $\text{cov}(v_k, v_k) = v_k \sim (0, R) = R$ and $\text{cov}(w_k, v_k) = 0$.

The objective is to design an estimator $\hat{x}_k$ that converges to $x_k$.

The estimation error is defined as $\hat{x}_{k+1} = x_{k+1} - \hat{x}_{k+1}$ and $P_{k+1} = \hat{x}_{k+1} \sim (0, P)$ is the covariance of the estimation error.

The Kalman filter works at each instant $k$, by estimating $\hat{x}_{k+1|k}$ and $P_{k+1|k}$ and corrects it on the next instant, $k + 1$.

The estimation at the instant $k$ is described as:

\[
\hat{x}_{k+1|k} = Ax_k + Bu_k
\]  

(11)

\[
P_{k+1|k} = AP_{k|k}A^T + G_kQG_k^T
\]  

(12)

and then, these are corrected at $k + 1$, according to (13) and (14).

\[
\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}(z_{k+1} - H\hat{x}_{k+1|k})
\]  

(13)

\[
P_{k+1|k+1} = [P_{k+1|k}^T + H^TRH]^{-1}
\]  

(14)

where $K$ is the Kalman gain and is defined as:

\[
K_{k+1} = P_{k+1|k+1}H^TR_k^{-1}
\]  

(15)

Although the Kalman filter is the optimal estimator for a system with white process and measurement noise, it can also be used in a system where noise is not completely white, with loss of performance.
2.4. Extended Kalman filter

The Kalman filter is a model based state estimator, that uses a linear system model, such as the one described on (10), but the semi-active suspension system is a non-linear system, because of the actuator saturation. To design a state observer in this case, the extended Kalman filter is used, which is basically a Kalman filter designed from a linearized system.

3. SIMULATION

3.1. Vehicle parameters

For the simulation, data from [1] was used. The vehicle parameters were considered only for the front quarter of the vehicle, and have numerical values as followed:

\[
\begin{align*}
    m_v &= 432.82 \text{ kg} \\
    m_w &= 40 \text{ kg} \\
    k &= 17200 \text{ N/m} \\
    k_w &= 200000 \text{ N/m}
\end{align*}
\]

The damper characteristics were also taken from [1], and the damper limits boundaries are:

\[
\begin{align*}
    \text{if } v_{wc} &\geq 0 \quad \begin{cases} 
    u &\leq 1480v_{wc} + 2852 \\
    u &\leq 3400v_{wc} + 1700 \\
    u &\leq 13500v_{wc} \\
    u &\geq 1500v_{wc} - 540 \\
    u &\geq 600v_{wc}
\end{cases} \\
    \text{if } v_{wc} &< 0 \quad \begin{cases} 
    u &\leq 600v_{wc} \\
    u &\leq 1000v_{wc} + 400 \\
    u &\geq 13500v_{wc} \\
    u &\geq 3000v_{wc} - 970 \\
    u &\geq 1200v_{wc} - 2050
    \end{cases}
\end{align*}
\]

The quarter-car in state space form, obtained from the dynamic equation (1) is:

\[
A = \begin{bmatrix}
    0 & 1 & 0 & 0 \\
    -39.7 & 0 & 39.7 & 0 \\
    0 & 0 & 0 & 1 \\
    430 & 0 & -5430 & 0
\end{bmatrix} \quad B_1 = \begin{bmatrix}
    0 \\
    0 \\
    4500
\end{bmatrix} \quad B_2 = \begin{bmatrix}
    0.0023 \\
    0 \\
    -0.0250
\end{bmatrix}
\]

3.2. \(\mathcal{H}_\infty\) design

The \(\mathcal{H}_\infty\) controller was designed with the use of MATLAB software. The force exerted by the damper was used as the control input, and the road input was considered the disturbance. Then, \(w, z, y\) and \(u\) were defined in (18).

\[
w = [d]^T \quad z = [\dot{z}_c] \quad y = [x_1, x_2, x_3, x_4]^T, \quad u = u(\beta, v_{wc})
\]

The choice of \(y\) as the full state vector was made since an observer is needed anyway to estimate the relative velocity of the damper \((v_{wc} = \dot{z}_w - \dot{z}_c)\). Also, the controlled system was shown to yield better results when using full state feedback, even when considering observer estimation errors. The performance vector \(z\) was chosen so that the chassis acceleration (eg. comfort index) is the variable to be optimized.

The target value of \(\gamma\) was chosen to be able to find the optimal controller that minimizes the acceleration, while having the road roughness as the disturbance. In this way, it was chosen \(\gamma = 40\). The high value of \(\gamma\) is directly related to the high gain between the road disturbance and the acceleration. Because of the inherent saturation characteristic of the semi-active damper, a lower target \(\gamma\) would lead to unfeasible control forces, which would then lead to saturation, resulting in a degraded performance. So the value of the target \(\gamma\) was chosen as to keep control forces low, while assuring a good performance.

3.3. Evaluation parameters

To evaluate the performance of the control system, quantitative methods of evaluation will be used. The controlled system will be compared with a passive system, with a damping constant \(\beta = 2000\).

The passenger comfort is directly related to the chassis acceleration. To better represent this parameter, its RMS value will be used, normalized by the gravity [14]. The relation is represented in equation (19).

\[
P_{zc} = \frac{z_{c,RMS}}{g} = \sqrt{\frac{1}{\tau} \int_0^\tau \left(\frac{\ddot{z}_c(t)}{g}\right)^2 dt}
\]

where \(\tau\) is the signal duration, in seconds and \(g\) is the gravity. Safety and handling depend on the contact forces between the tire and the road [14]. These can be defined as:

\[
F_{sw} = k_w(z_w - \dot{z}_r) + \beta_w(z_w - \dot{z}_r)
\]

This value is then normalized by the static forces acting in the vehicle (e.g. the vehicle and wheel weights). The handling evaluation parameter is completely defined in equation (21)

\[
P_{F_s} = \frac{F_{s,RMS}}{F_{s,stat}} = \sqrt{\frac{1}{\tau} \int_0^\tau \left(\frac{F_{s,w}(t)}{F_{s,stat}}\right)^2 dt}
\]

where \(\tau\) is the signal duration, in seconds.

In this work, the main objective was to increase comfort while still keeping good road handling characteristics. This translates into reducing the vertical chassis acceleration, \(\ddot{z}_c\), while keeping a low tire-road force interaction.
3.4. Road profiles

Road profiles that mimic real road situations will be used to test the controlled system, in comparison with the passive one. Two profiles will be used:

- English track: very rough and demanding road, with unevenly spaced holes;
- Drain well: road profile containing a high amplitude negative impulse.

The road profiles represent a road roughness amplitude in function of time, assuming a constant vehicle horizontal speed. Both the profiles have a duration of 14 seconds, with a horizontal speed 30 km/h for the drain well profile and 60 km/h for the English track profile. They are represented on Figures 4 and 5.

![Figure 4 – Drain well road profile with a constant horizontal speed of 30 km/h](image)

![Figure 5 – English track road profile with a constant horizontal speed of 60 km/h](image)

3.5. Kalman filter design

To be able to estimate the relative velocity $v_{wc}$, as well as minimize the number of sensors used in the system, the extended Kalman filter was designed, linearizing the system by considering a passive suspension system. The objective was to design an estimator that, given one noisy measurement (in the case of our system, the acceleration $\ddot{z}$ was chosen) could estimate all four state variables.

The Kalman filter is the optimal estimator when there is only white process and measurement noise, which is not the case of our system. While the assumption of white measurement noise can be true, the same cannot be considered for the process noise. To design the filter, white noise measurement noise was assumed, and multiple covariances for the process noise were assumed (considering the road profile covariances) and a filter was designed for each covariance, resulting in multiple designed filters. Then the filter with the least estimation error was picked as the optimal estimator to be used in the controlled system.

3.6. Simulation schematics

The system was implemented using Simulink. The quarter-car suspension system was used, as well as the Kalman filter and the $H_\infty$ controller. To compute the real damper force, a two stage configuration was used. On the first stage, the control force provided by the $H_\infty$ controller was clipped by a saturation block that uses the relative velocity estimate provided by the Kalman filter. The resulting clipped force is then used in a damper model that computes the real damper force based on the actual value of $v_{wc}$. The block diagram for the controlled system, using the state estimator is represented in Figure 6.

![Figure 6 – Simulation schematics for the $H_\infty$ controlled semi-active suspension system using a Kalman filter to estimate the system states](image)
Well road profiles. For the controlled systems, it was considered that only the vertical acceleration \( \ddot{z}_c \) was measured. To better represent a real sensor, Gaussian and biased noise were added to the acceleration measurement, with the Gaussian noise having zero mean and a covariance of around 1% of the RMS values of the acceleration, while the bias was valued around 0.1% of the acceleration value. A small time delay of \( 2 \times 10^{-6} \) seconds was also used, to simply represent the sensor dynamics.

On Figures 7 and 8, part of the simulation results are shown. The road profile was plotted together with the chassis acceleration of both systems, passive and with semi-active control. To better visualize the difference between the two systems, the RMS values of \( \ddot{z}_c \) normalized by gravity (as introduced in (19)) were plotted on Figure 9.

The frequency response of both systems was plotted on Figure 10, where the response between road disturbance and acceleration is shown, and on Figure 11, where the response between the road-tire interaction and the road disturbance is shown. Figures 9 and 10 show the improvement between the controlled system and the passive one. The \( \mathcal{H}_\infty \) controlled system has a better overall performance. Even though there is use of a state estimator to perform the state feedback to the controller, there is good performance. Regarding the acceleration minimization, the semi-active system performed very well, having reduced acceleration when compared with the passive system. Although there is improvement of the handling properties at low frequencies, as it can be seen on Figure 11, some degree of degradation is also present, around the resonance peak. This confirms the trade-off between comfort and handling properties.

![Figure 7](image1.png)

**Figure 7** – Top: Drain Well road profile; Bottom: Acceleration comparison between the \( \mathcal{H}_\infty \) control (—) and Passive system (—)

![Figure 8](image2.png)

**Figure 8** – Top: Drain Well road profile; Bottom: Comparison between the \( \mathcal{H}_\infty \) control (—) and Passive system (—)

![Figure 9](image3.png)

**Figure 9** – Comfort indexes for the controlled and passive systems - \( \mathcal{H}_\infty \) control (light grey); Passive system (dark grey)

4. CONCLUSIONS

In this paper, the semi-active suspension system was investigated using the \( \mathcal{H}_\infty \) control technique. The goal of one measurement sensor per quarter-car was also proposed, with the objective of reducing implementation costs. The performance of the controlled system was compared with that of a passive system, using road profiles that have the characteristics of real roads, and the performances were measured using quantitative methods of evaluation. The proposed system showed a good performance, specially when regarding the reduction of vertical acceleration, while showing small degradations regarding the road handling characteristics.

REFERENCES

[1] M. Canale, M. Milanese, and C. Novara. Semi-active suspension control using fast model-


