NONLINEAR MECHANICAL SYSTEM IDENTIFICATION USING DISCRETE-TIME
VOLterra MODELS AND KAutz FILTER

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Abstract: The present paper is concerned with the non-
parametric identification of mechanical systems with mild
nonlinearities using a Wiener-Volterra model. The paper
employs some classical developed results to describe the
discrete-time Volterra models using orthonormal Kautz func-
tions. If the two parameters of these filter sets, associated
with a Volterra series truncation, are properly designed, the
number of parameters needed to represent the Volterra ker-
nels is drastically reduced. Numerical tests illustrate the
results by detecting the first-order and the second-order
Volterra kernels.

Keywords: Volterra series, Kautz filter, orthogonal basis.

1. INTRODUCTION

The impulse response function (IRF) is often used in
linear system analysis through convolution integrals. If the
IRF is known, it is possible to simulate any response of
the linear system caused by an arbitrary input signal. The
nonlinear counterpart is also true, and it involves the use of
Volterra functionals. The main idea is to perform a multi-
dimensional convolution using high order kernels charac-
teristic of Volterra models. The Volterra series is com-
posed of multiple convolutions, that is to say, a filtering set
with a linear term (first kernel) and nonlinear kernels. For
this reason, the nonparametric Volterra models has been the
subject of several papers and books, for example [8] and [2].
The Volterra series has also been applied to the parametric
identification problem as discussed in the work [4].

Most papers to date use an extension of the frequency re-
response function (FRF) to multiples orders, namely high order
frequency response functions (HOFRF) or Volterra FRFs [5].
However, in the formulation of the present paper the Volterra
series are modeled directly in the time-domain. There are
several methods available to identify the Volterra kernels, for
instance interpolation techniques [6], associated linear equa-
tions [13], and the harmonic probing method [1] and [14].
One drawback of Volterra models is the large number of un-
known parameters in the power series. Despite these disad-
vantages, Tawfiq and Vinh [12] concluded in their work that
the Volterra series has received many unfounded criticisms.

A classical way to reduced the number of required pa-
rameters is to use an orthonormal basis. The Volterra model
is basically a Wiener model, where the terms are described
by orthogonal discrete-time filters \( z^{-1} \), where \( z \) is the com-
plex variable. Classical orthogonal bases are the Fourier
series and Legendre or Chebyshev polynomials. However,
there are other more effective orthogonal filters that enable
us to drastically reduce the convergence limits. These filters
contain information combining the mathematical functions
of the orthogonal filter with the real system [9]. The La-
guerre filter is an interesting example of an orthogonal filter
with poles that represent the dominant dynamic characteris-
tics [3]. However, in order to describe vibrating systems, the
Kautz filter is better since it contains complex poles to rep-
resent the second order dynamics [15]. The Wiener-Volterra
model with a Kautz filter is referred to in the present paper as
the Kautz-Volterra model. The goal of this paper is to eval-
uate the performance of the Kautz filter to describe the or-
thonormal basis of the discrete-time Wiener-Volterra models
in order to identify nonlinear mechanical systems. Numerical
simulations are presented to illustrate the proposed method-
ology.

2. VOLterra MODELS

The responses \( y(t) \) of a nonlinear dynamical system can
be written as

\[
y(t) = y_{\text{lin}}(t) + y_{\text{quad}}(t) + y_{\text{cub}}(t) + \cdots,
\]

where \( y_{\text{lin}}(t) \), \( y_{\text{quad}}(t) \), \cdots are the linear and nonlinear
contributions to the response \( y(t) \). If one knows the input-
output signals from experimental tests, then Eq. (1) can be
rewritten for each discrete-time sample \( k \)

\[
y(k) = y_{\text{lin}}(k) + y_{\text{quad}}(k) + y_{\text{cub}}(k) + \cdots.
\]
Equation (2) can be replaced by the discrete-time Volterra series with the following equation [8]

\[ y(k) = \sum_{n_1=-\infty}^{+\infty} h_1(n_1) u(k-n_1) + \sum_{n_1=-\infty}^{+\infty} \sum_{n_2=-\infty}^{+\infty} h_2(n_1, n_2) u(k-n_1) u(k-n_2) + \sum_{n_1=-\infty}^{+\infty} \sum_{n_2=-\infty}^{+\infty} \sum_{n_3=-\infty}^{+\infty} h_3(n_1, n_2, n_3) u(k-n_1) u(k-n_2) u(k-n_3), \]

where the terms \( h_1(n_1), h_2(n_1, n_2), \cdots \) are the Volterra kernels. The kernel \( h_1(n_1) \) is the classical linear contribution and is equivalent to the impulse response function.

A practical disadvantage in using the Volterra series is the large number of terms required to truncate the infinite power series and the large number of samples that is usually required. A reason for this is due to the fact that the nonlinearities have been modeled using only the input signals (excitation forces) \( u(k) \) while the feedback of the output times-series signals \( y(k) \) are neglected. The regression in the output signal \( y(k) \) represents the dynamics which are reflected in complex poles. The Volterra kernels have some interesting mathematical properties:

1. \( h_n(n_1, n_2, \cdots, n_n) \) can be written in a symmetric way.
2. \( \lim_{n_1 \to -\infty} h_n(n_1, n_2, \cdots, n_n) = 0 \) to \( l = 1, 2, \cdots, n. \)
3. \( h_n(n_1, n_2, \cdots, n_n) = 0 \) to \( \forall n_i < 0. \)

Equation (3) can be truncated to the finite terms \( N_1 \) and \( N_2. \) Assuming that the system is causal with only second order nonlinear terms, then

\[ y(k) = \sum_{n_1=0}^{N_1} h_1(n_1) u(k-n_1) + \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} h_2(n_1, n_2) u(k-n_1) u(k-n_2). \] (4)

It is also possible to express the kernels in a symmetric way, in which case \( h_2(n_1, n_2) = h_2(n_2, n_1). \) The general relationship yielding a symmetric kernel \( \text{sym} \) from an asymmetric kernel \( \text{asym} \) is [8]

\[ h_n(n_1, \cdots, n_n)_{\text{sym}} = \frac{1}{n!} \sum_{\pi(.)} h_n^{\text{asym}}(n_{\pi(1)}, \cdots, n_{\pi(n)}), \] (5)

where \( \pi(.) \) is the totality of possible permutations of the integer numbers \( 1, 2, \cdots, n. \) Equation (4) can then be rewritten again taking into account the symmetric kernel property

\[ y(k) = \sum_{n_1=0}^{N_1} h_1(n_1) u(k-n_1) + \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} h_2(n_1, n_2) u(k-n_1) u(k-n_2). \] (6)

Equation (6) is separated into linear and nonlinear contributions of Eq. (2) in the following way

\[ y_{\text{lin}}(k) = \sum_{n_1=0}^{N_1} h_1(n_1) u(k-n_1), \] (7)

\[ y_{\text{quad}}(k) = \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} h_2(n_1, n_2) u(k-n_1) u(k-n_2). \] (8)

Unfortunately it is generally very difficult to directly identify the kernels \( h_1(n_1) \) and \( h_2(n_1, n_2) \) from the experimental input-output signals if a nominal mathematical model of the structure is not available, since:

1. It is very difficult to find the Volterra kernels in a complex structure by analytical methods.
2. A large number \( N_1, N_2, \cdots \) is necessary to describe the kernels \( h_1(n_1) \) and \( h_2(n_1, n_2). \)

One possible way to estimate the kernels is by solving the following least squares (LS) problem

\[ y(k) = \Theta^T \lambda'(k), \] (9)

where \( \Theta \) is the parameter vector to be identified. The first and second order kernels are in the \( \Theta \) vector

\[ \Theta = \begin{bmatrix} h_1(0) & h_1(1) & \cdots & h_1(N_1) & h_2(0,0) & h_2(1,0) & h_2(1,1) & h_2(2,0) & h_2(2,1) & h_2(2,2) & h_2(3,0) & \cdots & h_2(N_2,0) & \ldots & h_2(N_2,N_2) \end{bmatrix}^T. \]

The regressive vector \( \lambda'(k) \) involving the input signal \( u(k) \) is written for each sample \( k \)

\[ \lambda'(k) = \begin{bmatrix} u(k) & u(k-1) & \cdots & u(k-N_1) & u(k) & u(k-1)u(k) & u(k-1)^2 & u(k-2)u(k) & u(k-2)u(k-1) & u(k-2)^2 & \cdots & u(k-N_2)u(k) & u(k-N_2)u(k-1) & \cdots & u(k-N_2)^2 \end{bmatrix}^T. \]
The number of parameters $N_p$ required to compute the kernels by solving the LS from Eq. (9) method is given by

$$N_p = \frac{1}{2} \left(1 + N_1 + \sum_{i=1}^{M} (N_i^1 + N_i^{i-1}) \right) - 1,$$  \hspace{1cm} (10)

where $M$ is the degree of nonlinearity. The high number is relative to the large number of points needed to describe the finite impulse response (FIR) filtering in Eq. (6). A classical application of the Volterra functionals is thus limited to structures with a small number of dofs.

3. WIENER-VOLterra MODELS

The Wiener-Volterra series is an orthogonal expansion of the classical Volterra series [10]. This mathematical approach is able to reduce the number of parameters $N_p$ required to evaluate the kernels. The basic idea is to represent the kernels $h_n(n_1, n_2, \ldots, n_n)$ with an orthogonal basis $\psi_{ij}(n_j)$, where $\psi_{ij}(n_j)$ is the impulse response of an orthogonal discrete filter $\Psi(z)$ and $z$ a complex variable

$$h_n(n_1, n_2, \ldots, n_n) \approx \sum_{i_1=1}^{M_1} \cdots \sum_{i_n=1}^{M_n} \alpha(i_1, \ldots, i_n) \prod_{j=1}^{n} \psi_{i_j}(n_j),$$  \hspace{1cm} (11)

It is assumed that the kernels are absolutely summable on $[0, \infty)$. The orthonormality property of $\psi_{ij}(n_j)$ is given by $\sum_{k=0}^{\infty} \psi_{ij}(k) \psi_{ij}(k) = \delta_{qq}$, where $\delta_{qq}$ is the Kronecker delta. The coefficients $\alpha(i_1, \ldots, i_n)$ are projections of the kernels onto the orthogonal basis $\psi_{ij}(n_j)$. The two kernels $h_1(n_1)$ and $h_2(n_1, n_2)$ are computed with Eq. (11) in the following way, assuming that the orthogonal kernel $\alpha(i_1, \ldots, i_n)$ is symmetric

$$h_1(n_1) = \sum_{i_1=1}^{M_1} \alpha(i_1) \psi_{i_1}(n_1),$$  \hspace{1cm} (12)

$$h_2(n_1, n_2) = \sum_{i_1=1}^{M_1} \sum_{i_2=1}^{M_2} \alpha(i_1, i_2) \psi_{i_1}(n_1) \psi_{i_2}(n_2),$$  \hspace{1cm} (13)

where $M_1$ and $M_2$ are the numbers of orthogonal filters used to approximate the kernels. Equations (12) and (13) can then be used to replace the kernels in Eq. (6)

$$y(k) = \sum_{i_1=1}^{M_1} \sum_{n_1=0}^{N_1} \alpha(i_1) \psi_{i_1}(n_1) u(k - n_1) + \sum_{i_2=1}^{M_2} \sum_{n_2=0}^{N_2} \alpha(i_2) \psi_{i_2}(n_2) u(k - n_2),$$  \hspace{1cm} (14)

Rewriting Eq. (14) yields

$$y(k) = \sum_{i_1=1}^{M_1} \sum_{n_1=0}^{N_1} \alpha(i_1) \psi_{i_1}(n_1) u(k - n_1) + \sum_{i_2=1}^{M_2} \sum_{n_2=0}^{N_2} \alpha(i_2) \psi_{i_2}(n_2) u(k - n_2).$$  \hspace{1cm} (15)

Hence, the input signals $u(k)$ in the Wiener-Volterra model described in Eq. (15) are filtered by discrete filters $\Psi(z)$ with impulse responses given by $\psi(k)$. It is now possible to write the following equations

$$l_1(k) = \sum_{n_1=0}^{N_1} \psi_{i_1}(n_1) u(k - n_1),$$  \hspace{1cm} (16)

$$l_2(k) = \sum_{n_2=0}^{N_2} \psi_{i_2}(n_2) u(k - n_2),$$  \hspace{1cm} (17)

where $\varepsilon = max(N_1, N_2)$. Equations (16) and (17) are substituted into Eq. (15). Finally, the Wiener-Volterra discrete-time model is obtained

$$y(k) = \sum_{i_1=1}^{M_1} \alpha(i_1) l_1(k) + \sum_{i_2=1}^{M_2} \alpha(i_2) l_2(k).$$  \hspace{1cm} (18)

The advantage of Eq. (18) compared to the classical Volterra model in Eq. (6) is the relatively small number of parameters to be identified in this new configuration. In Eq. (18), one must estimate the orthogonal kernels $\alpha(i_1)$ and $\alpha(i_1, i_2)$ with the number of parameters given by

$$N_w = \frac{1}{2} \left(1 + M_1 + \sum_{i=1}^{M} (M_i^1 + M_i^{i-1}) \right) - 1$$  \hspace{1cm} (19)

It is very important to observe that if $M_1$ and $M_2$ are smaller than $N_1$ and $N_2$, consequently $N_w < N_p$. It is thus easier to identify the orthogonal kernels $\alpha(i_1)$ and $\alpha(i_1, i_2)$ than their counterparts $h_1(n_1)$ and $h_2(n_1, n_2)$. Once the orthogonal kernels are obtained, Eq. (12) and (13) are used to compute the kernels in the physical space.

The general expression of the Volterra series in Eq. (3) can now be replaced by its counterpart using the orthogonal functions

$$y(k) = \sum_{m=1}^{M} \sum_{i_1=1}^{N_1} \cdots \sum_{i_m=1}^{N_m} \alpha(i_1, \ldots, i_m) \prod_{j=1}^{m} l_{i_j}(k)$$  \hspace{1cm} (20)

Eq. (9) is then used to solve the LS problem where the vector of parameters is given by

$$\Theta = \begin{bmatrix} \alpha(1) & \ldots & \alpha(M_1) & \alpha(1,1) & \alpha(2,1) \\ \alpha(2,2) & \alpha(3,1) & \ldots & \alpha(M_2, M_2) \end{bmatrix}^T.$$  \hspace{1cm} (21)
The regressive vector $\lambda'(k)$ is written by considering the input signals $u(k)$ filtered by the orthogonal filters

$$
\lambda'(k) = \begin{bmatrix} l_1(k) & l_2(k) & \cdots & l_{M_1}(k) \\
l_1(k)^2 & l_2(k)l_1(k) & \cdots & l_2(k)^2 \\
l_3(k)l_2(k) & l_3(k)^2 & \cdots & l_{M_2}(k)^2 \\
\end{bmatrix}^T
$$

(22)

4. KAUTZ FILTER

Several types of orthogonal functions $\psi_i(n_j)$ can be used to represent the kernels $h_1(n_1)$ and $h_2(n_1, n_2)$, for instance the polynomials of Chebyshev or Legendre. However, these functions are not directly related to the system dynamics via difference equations. Thus, the number of expansions in Eq. (12) and (13) remains high.

Fortunately, the Kautz filters are very effective in representing the orthogonal kernels. The Kautz filters contain poles with information on the dominant dynamics of the system [15] and [9]. The poles in a Kautz filter are thus able to decrease the number of parameters needed to estimate the kernels. Here $N$ is written by considering the orthogonal filters $\Psi$. The Kautz filter $\Psi_n(z)$ has a pair of complex poles $\beta = \sigma + j\omega$ and $\bar{\beta} = \sigma - j\omega$. These poles are given by two real scalar values $b$ and $c$, where $|b| < 1$ and $|c| < 1$ for stable poles. This filter constitutes a good generalization of the second-order dynamics of a vibrating system.

It remains to define the poles $\beta$ and $\bar{\beta}$ used to build the filters. The authors in [7] proposed a procedure for estimating the poles and the kernels $\alpha(i_1)$ and $\alpha(i_1, i_2)$ simultaneously in an iterative way. Meanwhile, in structural dynamics the linear structural matrices $M$, $C$ and $K$ are often available and this information can be used to estimate the Kautz poles in the $z$ plan.

The elements in a set of Kautz filters are given by [11] [9], [15] and [7]

$$
\Psi_{2n}(z) = \frac{\sqrt{(1 - c^2)(1 - b^2)}z}{z^2 + b(c - 1)z - c}^{n-1} \left[ \frac{-cz^2 + b(c - 1)z + 1}{z^2 + b(c - 1)z - c} \right]^{n-1}
$$

(23)

and

$$
\Psi_{2n-1}(z) = \frac{\sqrt{1-c^2}z - b}{z^2 + b(c - 1)z - c}^{n-1} \left[ \frac{-cz^2 + b(c - 1)z + 1}{z^2 + b(c - 1)z - c} \right]^{n-1}
$$

(24)

where the constants $b$ and $c$ can be expressed as a function of the poles $\beta$ and $\bar{\beta}$

$$
b = \frac{(\beta + \bar{\beta})}{(1 + \beta \bar{\beta})}, \quad (25)
$$

$$
c = -\beta \bar{\beta}. \quad (26)
$$

Hence, by using Eq. (23) and Eq. (24) the input signals $u(t)$ are filtered to obtain the signals $l_{i_1}(k)$ et $l_{i_2}(k)$. The IRF of Eq. (23) and (24) are saved to estimate the kernels. Here the Wiener-Volterra model constructed with a Kautz filter is called of Kautz-Volterra model.

5. NUMERICAL RESULTS

5.1. single dof model

A single dof Duffing oscillator is used to illustrate the estimation of the Kautz-Volterra model. The equation of motion is written by

$$
m\ddot{x} + c\dot{x} + kx + f_{nl}(t) = F(t),
$$

(27)

where the parameters of the model are given by $m = 1$ kg, $c = 20$ N.s/m, $k = 1 \times 10^4$ N/m and $f_{nl}(t)$ is the nonlinear force

$$
f_{nl}(t) = k_2x^2 + k_3x^3,
$$

(28)

where the quadratic and cubic stiffness parameters are $k_2 = 1 \times 10^4$ N/m² and $k_3 = 1 \times 10^{10}$ N/m³, respectively. In order to obtain the responses, Eq. (27) is solved by the Newmark and Newton-Raphson method. The sampling frequency is set to $F_s = 200$ Hz with $N = 1024$ time samples. The natural frequency is $15.9$ Hz and the modal damping factor is $\xi = 0.1$.

The excitation signal $F(t)$ used is a white noise sequence band-limited to the 13-18 Hz range. The time series of inputs and outputs are shown in Fig. 1. The power spectral density (PSD) is estimated by Welch’s method with $N = 1024$ points, an overlap of 50% and a Hanning window, which are shown in Fig. 2 and 3. The presence of non-linearities in this system can be seen in the series of harmonics of the natural frequency in the PSD. Hence, while the system is excited in the 13-18 Hz range, its response also has a higher frequency content.

![Figure 1](image-url) - Random input-output time-series.

The input-output signals from Fig. 1 are used to estimate the Volterra kernels. The linear parameters $m$, $c$ and $k$ are assumed to be known. Hence, the pair of complex poles in the Laplace $s$ domain can be estimated by $s_{1,2} = -\xi\omega_n \pm j\omega_n \sqrt{1 - \xi^2}$. In the present example, these poles are given by $s_{1,2} = -10 \pm 99.5j$. Meanwhile, since the Kautz filter is
a discrete-time system, the equivalent pole in the z domain is obtained from the relation $\beta = e^{sT}$ where $T = 1/F_s$. The discrete time poles are given by $\beta = 0.83 \pm 0.45j$.

The number of filters used to estimate the first orthogonal kernel $\alpha(i_1)$ is $M_1 = 4$ while $M_2 = 12$ filters are used to compute the second orthogonal kernel $\alpha(i_1, i_2)$. These values have been chosen by a trial-and-error procedure based on convergence in the series. The number of parameters to be identified is given by Eq. (19) with $N_w = 82$ in the present case. It is assumed that the kernels are null after $N_1 = N_2 = 120$ time samples. If Eq. (10) was used with a classical Volterra series, then $N_p = 7380$ parameters would need to be estimated, thus illustrating the significant gain obtained with a Kautz filter. It is important to notice that a 3rd order kernel $h_3(n_1, n_2, n_3)$ would be difficult to evaluate without a Wiener model. The choice of the Kautz poles based on the linear portion of the system dynamics should only be done if the nonlinear system is very simple as the present example made.

After obtaining the orthogonal kernel $\alpha(i_1)$, the physical kernel $h_1(n_1)$ can be found by projection in Eq. (12) once the impulse response of Kautz filter $\psi_{j_1}(n_j)$ is known. The result of the first kernel identification is presented in Fig. 4. The theoretical response $h_1(n_1)$ is also plotted for reference purposes.

The second order kernel $h_2(n_1, n_2)$ identified by the set of Kautz filter is shown in the Fig. 5. The number of filters employed $M_1$ and $M_2$ to obtain the kernels has a significant influence on the non-parametric identification. The values of $\beta$ and $\bar{\beta}$ are also responsible for the different results. In the simulation, it was assumed that the second kernel $h_2(n_1, n_2)$ is symmetric and has the same dynamics (poles) as the first kernel $h_1(n_1)$.

**5.2. Multiple dof model**

The method will now be applied to the two dof system shown in Fig. 6. The linear parameters are $m_1 = m_2 = 1$ kg, $k_1 = 1$ N/m, $k_2 = 15$ N/m and $k_3 = 1$ N/m. The stiffness of the nonlinear cubic spring is given by $k_{nl} = 15$ N/m$^3$ and the sampling rate is set to 20 Hz with 2048 samples. The excitation signal is a white noise with 0.5 N filtered by a 12th
order Butterworth digital filter with a cutoff frequency of 0.5 Hz. The input signal is applied to the first mass and only excites the first equivalent linear mode at 0.156 Hz. The simulated responses are obtained by numerical integration using the Newmark and Newton-Raphson method. Only the displacements of the first mass are assumed to be known. Fig. 7 shows the simulated input-output signals.

The pair of poles used in the $s$ domain are chosen as a function of the natural frequency $\omega_n$ and the modal damping factor $\xi$ of the equivalent linear model. If the linear matrices $M$, $C$ and $K$ of the model are assumed to be known then it is easy to obtain the values $s_1 = -0.1 \pm 0.995j$ and $s_2 = -0.3 \pm 2.985j$. However, the Kautz filters uses the values in the $z$ domain. Their transformation yields the discrete poles $z_1 = 0.9939 \pm 0.0495j$ et $z_2 = 0.9473 \pm 0.2703j$. These poles are used to describe the two equivalent linear models, but the Kautz filter in the present paper is used to describe only one linear mode. It is thus necessary to choose one mode that is representative of only one equivalent mode. The pair of discrete poles $z = 0.9742 \pm 0.1465j$ is found to be a reasonable choice to obtain a good estimate.

The number of filters used to estimate the first kernel $h_1(n_1)$ was set to $M_1 = 4$. Meanwhile, $M_2 = 2$ was used to identify the second kernel $h_2(n_1, n_2)$. The number of parameters to be identified is given by Eq. (19) with $N_w = 7$ in the present case. This number is related to the number of kernel parameters in the Kautz domain, given by $\alpha_1(i_1)$ and $\alpha_2(i_1, i_2)$. It is also assumed that the second kernel is symmetric and has the same dynamics as the first order kernel. Figures 8 and 9 show the results obtained.

The pair of poles used in the $s$ domain are chosen as a function of the natural frequency $\omega_n$ and the modal damping factor $\xi$ of the equivalent linear model. If the linear matrices $M$, $C$ and $K$ of the model are assumed to be known then it is easy to obtain the values $s_1 = -0.1 \pm 0.995j$ and $s_2 = -0.3 \pm 2.985j$. However, the Kautz filters uses the values in the $z$ domain. Their transformation yields the discrete poles $z_1 = 0.9939 \pm 0.0495j$ et $z_2 = 0.9473 \pm 0.2703j$. These poles are used to describe the two equivalent linear models, but the Kautz filter in the present paper is used to describe only one linear mode. It is thus necessary to choose one mode that is representative of only one equivalent mode. The pair of discrete poles $z = 0.9742 \pm 0.1465j$ is found to be a reasonable choice to obtain a good estimate.

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The final remarks

The paper presented an approach for calculating the discrete-time Volterra kernels from input-output time series signals. The method is based on a Wiener-Volterra series expressed in a Kautz filter basis. The number of parameters to estimate in this new configuration is dramatically smaller in comparison to a classical Volterra model. The reason is due to the fact that the Kautz filter has a pair of complex poles which accurately reflects the dominant dynamics of the nonlinear system.

REFERENCES


