SOME ASPECTS AND APPLICATIONS OF STATE OBSERVERS METHODOLOGY FOR CRACK DETECTION, LOCALIZATION AND EVALUATION IN MECHANICAL CONTINUOUS SYSTEMS

Edson Luiz Valverde Castilho Filho¹, Vinícius Fernandes², Gilberto Pechoto de Melo³

¹ Universidade Estadual Paulista “Júlio de Mesquita Filho”, Ilha Solteira, Brasil, edslufil@aluno.feis.unesp.br
² Universidade Estadual Paulista “Júlio de Mesquita Filho”, Ilha Solteira, Brasil, viniciusfer@aluno.feis.unesp.br
³ Universidades Estaduais Paulista “Júlio de Mesquita Filho”, Ilha Solteira, Brasil, gilberto@dem.feis.unesp.br

Abstract: The main purpose is to develop a new tool for fault diagnosis in continuous systems. A methodology to generate banks of state observers associated to a crack model was created in order to analyze the fault progress. A computational routine simulates the system that have been modeled trough the Finite Element Method, and a system of a vibrating cantilever beam, with displacement and velocity monitored, were used to do the experimental validation of the method and evaluate its applicability.

Keywords: State Observer, Crack, Continuous Systems.

1. PURPOSE

The purpose of this work is the study of a new mathematical model to discretize cracks at continuous mechanical systems applying it at a computational simulation using the methodology of State Observers validating it experimentally to evaluate its behavior applied at a real system.

2. THEME JUSTIFICATION

Lately, new techniques of fault detection and localization at mechanical systems dynamically loaded have been developed to attend the industry demand caused by the technology progress.

Even the tools for theoretical analysis of dynamic systems being sophisticated, there are great difficulties at the prediction of the dynamic behavior of some structural compounds and at the fault diagnosis, caused by the inaccuracy of the theoretical model, or caused by the difficulty of measuring some state variables.

The methodology of state observers is perfectly inserted on this reality, because its capability of estimate the state variables of a system based on the measurement of the output and control variables. The methodology becomes more attractive because it makes possible the reconstruction of the states where the measurement is hard or just impossible, detecting fails at points that are not available to be measured and monitoring then trough the reconstruction of its states.

Because of the magnitude of its effects, the crack nucleation or propagation demands essential care at mechanical systems. Knowing that this kind of fault can appear with the deterioration caused by vibrations and dynamical conditions, it becomes an excellent object for studying the use of the State Observers methodology to detect, locate and evaluate cracks conditions.

3. THEORETICAL FUNDAMENTALS

Basically a state observer estimates the state variables using as base the measurements of the output and control variables. This technique consists in a method capable of reconstruct the states of a system where the measurement is compromised or impossible, being able to detect faults at these points without the knowledge of its measurements [1]. That can also be monitored with the reconstruction of its states. In 1966, Luenberger demonstrated in his work that if a system is linear, its state array can approximately be reconstructed trough the project of an observer and, in 1971, the same author introduces the concept of many types of observers, as example, the Identity Observer [2] which uses a Linear Transformation of the data acquired from the output of the system to compare to the observers results. Despite the definition be relatively old, the proposed observers still being utilized, being theme of a great number of researches.

In 1990 was established the stiffness matrix of the cracked element and also studied the motion equation of a cracked cantilever beam [3], using the equations provided by the fracture mechanic, turning possible modeling a cracked system. In 1995 was created a model of a beam using the Finite Element Method which can apply the Qian’s model of the crack, turning possible to study a new form to detect this kind of problem [4]. Today the study of crack detection in mechanical systems lies over the developing of new observers and in the research and construction of crack models, making, then, more accurate the predictions that the simulations can bring.

3.1. State Observers

Since 1964, the observers have been performing part of numerous projects of control systems, where a small part
SOME ASPECTS AND APPLICATIONS OF STATE OBSERVERS METHODOLOGY FOR CRACK DETECTION, LOCALIZATION AND EVALUATION IN MECHANICAL CONTINUOUS SYSTEMS

Edson Luiz Valverde Castilho Filho, Vinicius Fernandes, Gilberto Pechoto de Melo

has been shown in an explicit way. The simplicity of its project and resolution make the state observer an attractive component of the project, because it can reconstruct the non measured states of the system [5].

A state observer for an original dynamic system with the state \{x(t)\}, \{y(t)\}, as the output and the input being \{u(t)\}, is an auxiliary dynamic system. In other words, it is a copy of the original system that has the same input of this system and has the capability of estimate the unknown system states from states that are known. The next figure shows this definition, considering \[L\] as the State Observer Matrix.

![Fig. 1. The State Observer definition.](image)

The construction of an observer is just possible if the original system is able to be observed or at least detectable. Differing from the system \[\dot{x}(t)\], that is physical, the system \[\dot{\hat{x}}(t)\] is something abstract and generated by a computer program.

There are a great number of kinds of state observers, but the identity observer had been chosen to the realization of the research, because it has good convergence and easy implementation.

### 3.1.1. Controllability and Observability

The concepts of controllability and observability are very important at the control study and at the estimation of dynamic systems. The concept of controllability has relationship with the possibility of the existence of a control law, because there are cases that are not possible to find a law where the system is stable considering a projected trajectory [1]. On a similar way, the concept of observability has relationship with the existence of an algorithm that is capable to estimate the state variables from the available variables.

A system is called completely controllable if at any instance, with a non restrict input, its possible to transfer the initial system to any other state in a finite interval of time, and if the system is controllable for any instance and initial state, the system is also called completely controllable.

A system is called observable in any instance if it’s possible to determinate the initial state from the response of the system in a finite interval of time. It implies that all the state variables have an influence at the response.

These concepts can be better illustrated with the figure 2:

![Fig. 2. Concepts of Controllability and Observability.](image)

Where:

- \(S_{co}\) = Subsystem completely observable and controllable.
- \(S_{o}\) = Subsystem completely observable but not controllable.
- \(S_{c}\) = Subsystem completely controllable but not observable.
- \(S_{u}\) = Subsystem not observable and not controllable.

### 3.1.2. Identity Observer

That is considered, for the description of the Identity Observer, the linear and time invariant system shown by equation (1):

\[
\begin{align*}
\dot{x}(t) &= [A]x(t) + [B]u(t) \\
y(t) &= [C_{me}]x(t) + [D]u(t)
\end{align*}
\]

(1)

Where \([A] \in \mathbb{R}^{nxn}\), \([B] \in \mathbb{R}^{nxp}\), \([C_{me}] \in \mathbb{R}^{kxn}\), \([D] \in \mathbb{R}^{kxp}\), \(n\) the order of the system, \(p\) the number of inputs \([u(t)]\) and \(k\) the number of outputs \([y(t)]\). Taking the system as completely observable [1].

An observer for a system like this is:

\[
\begin{align*}
\dot{x}(t) &= [A]x(t) + [B]u(t) + [L]\{\{y(t)\} - \bar{y}(t)\} \\
\bar{y}(t) &= [C_{me}]x(t)
\end{align*}
\]

(2)

And

\[
\bar{y}(t) = [C_{me}]\{x(t)\}
\]

(3)

Where, \([L]\) is the state observer matrix.

The estimation error for the state is:

\[
\{e(t)\} = \{\bar{x}(t)\} - \{x(t)\}
\]

(4)

And the estimation error at the output (residue):

\[
\{e(t)\} = \{\bar{y}(t)\} - \{y(t)\}
\]

(5)

Now, substituting the equations (1), (2) and (3) in (4) and (5), what is got is:
\[
\{e(t)\} = ([A] - [L][C_{mc}])[\{e(t)\}] + [L][D][u(t)]
\]  
(6)

And

\[
\{\dot{e}(t)\} = [C_{mc}][\{e(t)\}] - [D][\{u(t)\}]
\]  
(7)

Where, the expression \( \{\dot{e}(t)\} = d[\{e(t)\}] / dt \) represents the evolution of the error from the observer.

### 3.1.3. Global and Robust Observers

Two kinds of observers are used at the detection and localization of faults at dynamic systems. The global observer is the responsible for detect a possible fault at the system, while the robust observer is capable of locating the parameter that failed [2].

The global observer is nothing more than a copy of the original system. So, is possible to make a comparison of the collected parameters with the ones that have constructed by the global observer. If any difference appears between the behaviors of the curves can be concluded that the real system is failing.

From this information, the new focus is the search of the faulty parameter, constructing robust observers for every parameter that is able to fail. These observers are constructed with a gradual alteration on its dynamic matrix at the respective parameters for which they are robust. If the behavior of the robust observer of a determinate parameter gets close of the real behavior of the system, then that can be concluded that this parameter is failing [2].

This methodology can be employed for any mechanical system that is intended to control. The main idea is to construct a monitoring bank of robust observers to each parameter of the system, because they can constantly send the RMS differences between the obtained signals and the graphic generated by the observer, to a logic decision unit, that will judge if there is a fault at a parameter [6].

The figure below represents the functioning of a logic decision unit based on the information given by the observers.

**Fig. 3. Principle of a system monitoring with robust observers.**

### 3.2. Cracked Beams

Knowing the risks that one crack can produces inside a mechanical system, its occurrence and identification is indispensable for the structure health analysis. The crack position and its dimensions can be detected by the disturbance of the natural frequency and mode shapes of the system. When a beam is dynamically loaded, and it has a crack inside, this crack will open and close alternately, depending on the vibration direction, causing a variation of the physical parameters of the system, as an example, a stiffness variance.

The presence of a crack at a beam, according to Saint-Venant’s Principle, causes a neighborhood perturbation at the tension distribution. This perturbation is especially relevant when the crack is opened and determines a local reduction of the stiffness, so if the crack is closed, can be considered that there is no disturbances at the system.

When this kind of system is discretized by Finite Elements, it is necessary to take an essential care with the construction of its mass, stiffness and dumping matrices.

Making the assumption that with a crack there is no mass losses, can be concluded that the mass matrices won’t suffer any effect of the crack, because even the cracked element still have its mass matrix unaltered:

\[
[M] = \frac{mL}{420}
\]

\[
\begin{bmatrix}
156 & 22l & 54 & -13l \\
22l & 4l^2 & 13l & -3l^2 \\
54 & 13l & 156 & -22l \\
-13l & 54 & -3l^2 & 4l^2
\end{bmatrix}
\]  
(8)

The dumping matrix is very hard to be obtained with theoretical procedures, so it is considered structural and obtained with the assumption that the beam is a single degree system. So, using some beam parameters could be found the logarithm decrement (ξ) of the beam displacement behavior, and using the natural frequency of the system an equation for the equivalent dumping can be found:

\[
C_{eq} = 4\pi.m.f_n.\xi
\]  
(9)

Where \( f_n \) is the natural frequency of the system without fault and \( m \) is the beam mass. Placing the result at a 4x4 eye matrix can be obtained the dumping matrix for the element:

\[
[C] = C_{eq}
\]

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  
(10)
3.2.1. Stiffness Matrix of a Cracked Element

The biggest problem to determine how a cracked system can be described is the stiffness matrix. All the approximations that can be done come from a complex theory developed to be used at numerical methods. According to Saint-Venant’s Principle, the tension field is only affected at the adjacent region of the crack. So, the stiffness matrices of the elements, with the exception of the cracked element, can be considered unaltered under a certain limitation of the element size and it fits in the theory of Euler Bernoulli with hermitian function:

\[
[K] = \begin{bmatrix}
E_{bh}^3 & E_{bh}^3 & -E_{bh}^3 & E_{bh}^3 \\
E_{bh}^3 & 2l^2 & E_{bh}^3 & 2l^2 \\
-2l^2 & 3l & -E_{bh}^3 & 6l \\
E_{bh}^3 & -E_{bh}^3 & E_{bh}^3 & -E_{bh}^3 \\
2l^2 & 6l & 2l^2 & 3l
\end{bmatrix}
\] (11)

Because of the discontinuity of the deformation at the cracked element, it’s very hard to find an appropriated function to express, approximately, the potential elastic energy. The calculus of the additional tension energy has been deeply studied through the fracture mechanics. Then, the expression of the cracked element matrix \((k_u)\) is an explicit function of a lot of parameters, such as flexibility coefficients and cracks dimensions. However, the matrix can be writing as a relation of evaluation coefficients, where the coefficient for the condition for open crack \((\alpha)\) is tabulated. This coefficient is a function of the crack depth and the relation between height and length of the cracked element. It has direct influence at the stiffness matrix of the cracked element:

\[
k_{Trinca} = \alpha_1 \begin{bmatrix}
k_{11}α_2 & k_{12}α_2 & k_{13}α_2 & k_{14}α_2 \\
k_{12}α_2 & k_{22}α_3 & k_{23}α_2 & k_{24}α_4 \\
k_{13}α_2 & k_{23}α_2 & k_{33}α_2 & k_{34}α_2 \\
k_{14}α_2 & k_{24}α_4 & k_{34}α_2 & k_{44}α_3
\end{bmatrix}
\] (12)

Still, for each robust observer designed, there is a change in matrix stiffness in the position of the broken part, and reduced their values according to the proportional constant. Therefore, each new dynamic matrix built (13), should be recalculated the quadrant related to the stiffness matrix of the same (in Quadrant 3).

\[
[A]_{2m,2n} = \begin{bmatrix}
0_{m,n} & \cdots & [I]_{m,n} \\
\cdots & \cdots & \cdots
\end{bmatrix}
\] (13)

3.2.2. Motion Equation for the Element

The dynamic response of the beam in the intervals of time that the crack is closed can be considered, for simplicity, such as a beam without crack. This is because the crack interfaces interact with each other completely. Under the action of the force of excitation, the opening and closing of the crack will alternate against time.

The equation of motion of the cracked beam discretized by N finite elements and subjected to a vector of external excitation \(F(t)\) can be written as (14), where \(M\) is the mass matrix, \(C\) is the matrix of damping, \((K_u - γΔK)\) is the matrix of stiffness and \(ΔK\) is the matrix of stiffness. As definition \(γ = 1\) when the crack is open and \(γ = 0\) when the crack is closed.

\[
M\ddot{u}(t) + C\dot{u}(t) + (K_u - γΔK)u(t) = f(t)
\] (14)

Was considered \(γ = 1\), because as long as the crack remains closed \((γ = 0)\) the stiffness matrix is composed only by the portions where the crack is not considered, so at that moment there is no failure.

4. METHODOLOGY

4.1. Computational Simulations

For the suggested system, a coupled cantilever beam, were used a Finite Element Method with beam elements at an elastic foundation [4], obeying the Crack model described before. The beam is discretized in five elements and at one of them placed the crack. The cracked element was modeled with the open crack parameters during all the simulation because were found difficulties to implement the dynamic crack.

![Cracked Beam scheme with the Crack placed at the second element.](image)

For this system were simulated conditions of impulsive impact and harmonic excitation, and analyzed the results supplied by the State Observers through RMS differences between the two function curves.

A complete observation system with a Global Observer of the process and Robust Observers, dedicated to accompany the stiffness variation of each element, was used, locating the fault and evaluating the percentage of penetration of the crack in the beam.

The simulations were computationally developed at MATLAB® software in two steps: First was constructed a stiffness matrix creation algorithm through the study of the beam parameters, and after, with these results, a routine was developed with detecting, finding and evaluating faults as function, with graphical and numerical analysis.
4.2. Experimental Step

Experimentally, were assembled a system with a cantilever beam with the nearest characteristics of the simulated one, a mild steel beam coupled to an inertial table, with the measurement realized through an accelerometer positioned in the direction of its freedom degrees. The excitations are realized by an initial displacement, in the impulsive excitation case, and with an electro-mechanic oscillator, in the harmonic excitation case. With the measurements obtained by these installed accelerometers, was possible to compare, in real time, the experimental results with the simulated ones, using the State Observers projected for the system, creating then, a difference bank between the two signals for comparison. Were generated the function curves for both functions to compare the disparity between the real and the simulated graphs.

At the experimental step, the development of computational routines was completed with the creation of a routine at the SIMULINK (MATLAB®) environment that executes a real time reading of the signals provided by the acquisition system processing them, locating and evaluating the crack. The routine processes the detection, localization and evaluation of the fault at its measured state, presenting as final results a compilation of all system analysis brought by the reconstructions of the states by the Observers.

The experimental study was focused on the Harmonic Excitation, for time domain analysis of the permanent state of the system. For this, considering the modification that an accelerometer would make due to its weight, a piezoelectric sensor was used with a measurement system.

The Impulsive Excitation experiment was assembled with an initial displacement condition to the beam causing an impulse function force.

5. RESULTS

5.1. Simulation Results

The results were obtained considering previously that the crack position is already known and the study object is only the evaluation of the crack condition. However the fault localization isn’t too much different, because the same process is done, but now, using all percentage of penetration for all the stiffness parameters of each element. In this case the global observer detected the fault and the robust observers are placed just at one element, the cracked one.

For this simulation a mild steel cantilever beam was considered having the dimensions of length \(L=0.6\text{m}\), height \(h=12.5\times10^{-3}\text{m}\), width \(b=17.2\times10^{-3}\text{m}\), Elasticity modulus \(E=2.07\times10^{11}\text{N/m}\), density \(\rho=7850\text{kg/m}^3\) and Element length of \(l=0.12\text{m}\), being the initial condition for all the simulations a null displacement for all the elements. The time interval chosen was from 0 to 0.4 seconds for impulsive excitation and harmonic excitation, being both of them shared into 1024 points to plot.

Considering that each element has its own stiffness matrix (11), mass matrix (8) and dumping matrix (10), it’s necessary respect the coupling effects and the crack alterations at them. So the matrices are changed from its usual configuration.

As said before, the only alteration did to adjust the model to be more close to the real situation is the stiffness matrix for the system alteration, because there is no mass significant loss and the dumping is calculated from the Structural Dumping. The alteration is done by proportional parameters that depend on the nature of the crack, being searched and included at Robust Observers.

The mass (13), stiffness (14) and dumping (15) matrices of the beam without fault are described below, however, for each Robust Observer projected, there is an alteration at the stiffness matrix at the cracked element position, being its values reduced proportionally to its constants \(\alpha\). Reminding that the Element length \(l\) was calculated as the equal division of the beam length by the number of elements, and the Inertia Moment \(I\) calculated with the parameters \(h\) and \(b\).

\[
\begin{bmatrix}
156 & 221 & 54 & -131 & 0 & 0 & 0 & 0 & 0 & 0 \\
221 & 4l^2 & 13l - 3l^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
54 & 13l & 312 & 0 & 54 & -13l & 0 & 0 & 0 & 0 \\
-13l - 3l^2 & 0 & 8l^2 & 13l - 3l^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 54 & 13l & 312 & 0 & 54 & -13l & 0 & 0 \\
0 & 0 & -13l - 3l^2 & 0 & 8l^2 & 13l - 3l^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 54 & 13l & 312 & 0 & 54 & -13l \\
0 & 0 & 0 & 0 & -13l - 3l^2 & 0 & 8l^2 & 13l - 3l^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 54 & 13l & 156 & -22l & 4l^2 \\
0 & 0 & 0 & 0 & 0 & 0 & -13l - 3l^2 & -22l & 4l^2 & 0 \\
\end{bmatrix}
\]
For the realized computational simulations could be verified the detection and localization of the fault comparing the global system without fault and the global observer for the system calibration. The RMS difference of 10E-12 shows a curve coincidence. It means that, if the real system stay practically equal to the global observer (with no fault), there is no fault at the system.

The simulations were done by an impact force, described by a unitary impulse function (or not null initial velocity), and by a harmonic force, of periodic actuation and by an harmonic force, of periodic actuation and by a unitary impulse function (or not null initial velocity), there is no fault at the system.

The amplitude of 10N and a frequency of 100Hz.

\[
C_0 = \begin{bmatrix}
61 & -12 & 61 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
61 & 41 & -61 & 21 & 0 & 0 & 0 & 0 & 0 & 0 \\
-12 & -61 & 24 & 0 & -12 & 61 & 0 & 0 & 0 & 0 \\
61 & 21 & 0 & 81 & -61 & 21 & 0 & 0 & 0 & 0 \\
0 & 0 & -12 & -61 & 24 & 0 & -12 & 61 & 0 & 0 \\
0 & 0 & 61 & 21 & 0 & 81 & -61 & 21 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -12 & -61 & 24 & 0 & -12 \\
0 & 0 & 0 & 0 & 0 & 61 & 21 & -61 & 41 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
C_{i_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & C_{i_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & C_{i_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & C_{i_0} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & C_{i_0} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & C_{i_0} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & C_{i_0} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{i_0} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{i_0} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{i_0}
\end{bmatrix}
\]

\[
K = E \frac{1}{T}
\]

Table 1. Result of the RMS differences for an Impact Force Simulation

<table>
<thead>
<tr>
<th>Simulated System Without Fault</th>
<th>Global Obs.</th>
<th>9.0647e-12</th>
<th>Obsv. 5%</th>
<th>2.0342e-03</th>
<th>Obsv. 10%</th>
<th>6.1713e-03</th>
<th>Obsv. 15%</th>
<th>2.5082e-02</th>
<th>Obsv. 20%</th>
<th>5.1826e-02</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated 5% of Fault</td>
<td>Global Obs.</td>
<td>1.6981e-03</td>
<td>Obsv. 5%</td>
<td>1.9134e-11</td>
<td>Obsv. 10%</td>
<td>2.7802e-02</td>
<td>Obsv. 15%</td>
<td>4.0757e-02</td>
<td>Obsv. 20%</td>
<td>5.5858e-02</td>
</tr>
<tr>
<td>Simulated 10% of Fault</td>
<td>Global Obs.</td>
<td>1.0738e-01</td>
<td>Obsv. 5%</td>
<td>2.0236e-02</td>
<td>Obsv. 10%</td>
<td>3.5412e-02</td>
<td>Obsv. 15%</td>
<td>3.7526e-02</td>
<td>Obsv. 20%</td>
<td>7.6434e-12</td>
</tr>
</tbody>
</table>

Table 2. Result of the RMS differences for a Harmonic Force Simulation

<table>
<thead>
<tr>
<th>Simulated System Without Fault</th>
<th>Global Obs.</th>
<th>1.2028e-13</th>
<th>Obsv. 5%</th>
<th>2.6341e-02</th>
<th>Obsv. 10%</th>
<th>3.8246e-02</th>
<th>Obsv. 15%</th>
<th>9.7542e-03</th>
<th>Obsv. 20%</th>
<th>3.0129e-02</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated 5% of Fault</td>
<td>Global Obs.</td>
<td>2.9342e-02</td>
<td>Obsv. 5%</td>
<td>3.7426e-12</td>
<td>Obsv. 10%</td>
<td>3.1402e-03</td>
<td>Obsv. 15%</td>
<td>4.9484e-03</td>
<td>Obsv. 20%</td>
<td>5.9874e-02</td>
</tr>
<tr>
<td>Simulated 10% of Fault</td>
<td>Global Obs.</td>
<td>8.7346e-03</td>
<td>Obsv. 5%</td>
<td>9.6035e-02</td>
<td>Obsv. 10%</td>
<td>9.7826e-12</td>
<td>Obsv. 15%</td>
<td>1.9896e-03</td>
<td>Obsv. 20%</td>
<td>5.1765e-03</td>
</tr>
<tr>
<td>Simulated 15% of Fault</td>
<td>Global Obs.</td>
<td>4.6538e-03</td>
<td>Obsv. 5%</td>
<td>7.6985e-02</td>
<td>Obsv. 10%</td>
<td>5.7248e-03</td>
<td>Obsv. 15%</td>
<td>1.2645e-11</td>
<td>Obsv. 20%</td>
<td>2.4578e-02</td>
</tr>
</tbody>
</table>

Once the faults are put into the system, they are detected by a divergence between the curves already mentioned, and, through the action of the robust observers they are found and evaluated. Exemplifying, on the second line and second column of the Table 1, can be verified a detection of 5% of fault, so, the crack reach 5% of the total height of the element. At the sequence, were inserted faults varying from 5% to 5% until reaching 20% of height of the element, accompanied by the robust observers which identified them.

The graphical solution of the problem clarifies more didactically how the functioning of the State Observers is. At the figures below, can be observed a System excited harmonically with a crack size of 30% of the beam height.
As observed, there was a coincidence between the curves of the simulated system and the State Observers, projected to detect the respective size percentage of the crack, represented by the hatched elements, because the RMS difference values between the curves tends to zero, what show the efficiency of the State Observers bank that was used.

First the simulated system was compared to the global observer. The system wasn’t working in perfect conditions, then there was detected a divergence between the Observer and the Simulated System curves. Knowing about a irregularity of the system, the Robust Observers keep on looking after the faulty parameter, being then, founded by a superposition of the function curves.

Now, a system with a not null initial velocity (2.5 m/s), with a crack size of 30% of the beam height is simulated with the same intention. The same mechanism of detection is used here.
In both cases, the kinematical magnitude used to do the analysis was the velocity. It is common to use this variable of the process because the measurement equipment is an accelerometer which gives, as result of the measurement, the acceleration of the vibration and with the numerical integration of the curve, given by the own equipment, there is velocity as response.

5.2. Experimental Results

For the experimental analysis of harmonic excitation, a coupled beam discretized as Figure (4). The system was excited through a harmonic force with the same function used at the simulated one applied at the 3rd node. There were experimented different excitation frequencies, 5Hz, 10Hz, 20Hz, 50Hz, 100Hz e 200Hz, and also controlled the fault conditions using as parameter of control the height of the notch made at the beam to ensure the loose of stiffness.

The system was observed with a real time observation system as described before at SIMULINK (MATLAB®). Experimentally the good results were obtained for medium and high frequencies. The method showed itself incapable to detect the faults due to a negative influence of the mechanical response of the structure which causes an oscillation slower than the frequency of the force, what makes that the provided states given by the observers doesn’t be the same as the values gotten at the structure, turning difficult the acquiring of differences values between functions that are capable to localize the fault and evaluate it. The same phenomenon was observed at the natural frequencies of the structure. This conclusion can be proved through the next tables.

<table>
<thead>
<tr>
<th>Table 3. Result of the RMS differences for the cracked element at the experiment with the 5Hz Harmonic Force (Low Frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Real System Without Crack</strong></td>
</tr>
<tr>
<td>Global Obsv.</td>
</tr>
<tr>
<td>Obsv. 10%</td>
</tr>
<tr>
<td>Obsv. 20%</td>
</tr>
<tr>
<td>Obsv. 30%</td>
</tr>
<tr>
<td><strong>Real System With Crack (10%)</strong></td>
</tr>
<tr>
<td>Global Obsv.</td>
</tr>
<tr>
<td>Obsv. 10%</td>
</tr>
<tr>
<td>Obsv. 20%</td>
</tr>
<tr>
<td>Obsv. 30%</td>
</tr>
<tr>
<td><strong>Real System With Crack (20%)</strong></td>
</tr>
<tr>
<td>Global Obsv.</td>
</tr>
<tr>
<td>Obsv. 10%</td>
</tr>
<tr>
<td>Obsv. 20%</td>
</tr>
<tr>
<td>Obsv. 30%</td>
</tr>
<tr>
<td><strong>Real System With Crack (30%)</strong></td>
</tr>
<tr>
<td>Global Obsv.</td>
</tr>
<tr>
<td>Obsv. 10%</td>
</tr>
<tr>
<td>Obsv. 20%</td>
</tr>
<tr>
<td>Obsv. 30%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4. Result of the RMS differences for the cracked element at the experiment with the 20Hz Harmonic Force (Medium Frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Real System Without Crack</strong></td>
</tr>
<tr>
<td>Global Obsv.</td>
</tr>
<tr>
<td>Obsv. 10%</td>
</tr>
<tr>
<td>Obsv. 20%</td>
</tr>
<tr>
<td>Obsv. 30%</td>
</tr>
<tr>
<td><strong>Real System With Crack (10%)</strong></td>
</tr>
<tr>
<td>Global Obsv.</td>
</tr>
<tr>
<td>Obsv. 10%</td>
</tr>
<tr>
<td>Obsv. 20%</td>
</tr>
<tr>
<td>Obsv. 30%</td>
</tr>
<tr>
<td><strong>Real System With Crack (20%)</strong></td>
</tr>
<tr>
<td>Global Obsv.</td>
</tr>
<tr>
<td>Obsv. 10%</td>
</tr>
<tr>
<td>Obsv. 20%</td>
</tr>
<tr>
<td>Obsv. 30%</td>
</tr>
<tr>
<td><strong>Real System With Crack (30%)</strong></td>
</tr>
<tr>
<td>Global Obsv.</td>
</tr>
<tr>
<td>Obsv. 10%</td>
</tr>
<tr>
<td>Obsv. 20%</td>
</tr>
<tr>
<td>Obsv. 30%</td>
</tr>
</tbody>
</table>

As can be observed, at the first table, the experimented values for lower frequencies and lower crack percentages did not show differences values sufficiently big to indicate the fault. Graphically, there can be observed the kind of divergence that occurs when there is the comparison of the values gotten with the system excited with 5Hz and with 20Hz for the same crack percentage. The cases use the lower detectable crack length, with 10% of the beam width because they represent the most visible way this kind of occurrence.
This defect is not only caused by the effect of the low frequency, but also due to the values obtained from the Dynamic Matrix being very close to values of a system without fault, because the small percentage of crack depth, making the sum of the effects prejudicial to the observers work.

Generally the problems that can be modeled by this kind of methodology show the excitation frequency higher than 10Hz, considering that the research aims industrial applying. Then, there cannot be considered that this imperfection is a big trouble for the methodology. To prove this limit, next are shown the experimental results for a system excited with 10Hz and a crack depth of 10%.

### Table 5. Result of the RMS differences for the cracked element at the experiment with a periodic force with 10Hz frequency

<table>
<thead>
<tr>
<th>Real System Without Crack</th>
<th>Global Obsv.</th>
<th>Osv. 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9.5124e-02</td>
<td>2.6179e-01</td>
</tr>
<tr>
<td>Real System With Crack (10%)</td>
<td>3.3299e-01</td>
<td>8.7218e-03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Real System With Crack (20%)</th>
<th>Global Obsv.</th>
<th>Osv. 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.0976e-01</td>
<td>1.2986e-02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Real System With Crack (30%)</th>
<th>Global Obsv.</th>
<th>Osv. 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.834e-02</td>
<td>3.4231e-02</td>
</tr>
</tbody>
</table>

Fig. 15. Comparative Graphs of the Robust Observer to 10% depth of the Real System Excited with 5Hz and 20Hz

Fig. 7. Results of the Observation of the System Excited with a 10Hz frequency force and a crack depth of 10 percent. (The first one is the robust observer which located and evaluated the fault, and the second is the global observer of the process)
6. CONCLUSIONS

Through this work there was noticed that in the study realized to faulty continuous system, the localization of the faults is obtained with a large number of measurements done in the structure. The state observer technique uses fewer measurements with the reconstruction of the other states.

The computational analysis for the developed method have shown good results for the simulated system with a crack, what shows that the mathematic approaches used to model the system have generated results that are applicable at real systems.

In the experimental step, could be observed that only the robust state observer designated to a specific crack percentage can detect the irregularity presented, showing that the method not only detect and localizes the fault, but also can avoid the problem magnitude. However, there must be an essential care with the excitation frequency and its restrictions, because the observations system can conduct to an incorrect result in the cases that were specially studied.

The experimental results also show that, despite the magnitude orders have not great difference between them, the method can be applied at simple systems.

So, analyzing the obtained results through computational simulations and experimental validation, there can be concluded that the developed methodology to detection and localization of faults at continuous mechanical systems using fault models provided satisfactory results, showing the efficiency of the developed methodology.

ACKNOWLEDGMENTS

Our thanks go to our families for the comprehension and support, Prof. Dr. Gilberto Pechoto de Melo for orientation, and patience, Mechanical Engineering Department /FEIS for technical support and CNPq/PIBIC/UNESP that is supporting this work.

REFERENCES