We consider iterative solutions for the nonlinear equation
\[ M(\int_{-L}^{L} u(s) \, ds) u''(x) = f(x, u(x), u(-x)), \quad x \in [-L, L] \] and subject to the boundary condition
\[ u(-L) = u(L) = 0, \quad (2) \]
where \( M : \mathbb{R} \to \mathbb{R} \) and \( f : [-L, L] \times \mathbb{R}^2 \to \mathbb{R} \) are continuous functions with \( M \) satisfying:
\[ \exists \delta > 0 \text{ such that } M(s) \geq \delta \text{ for all } s \in \mathbb{R}. \quad (3) \]
These differential equations are also variations of the stationary form of the Kirchhoff equation
\[ u_{tt} - \left( c_0 + c_1 \int_{-L}^{L} |u_x|^2 \, dx \right) u_{xx} = 0, \]
which is a classical nonlinear model for the study of the vibrations of elastic strings. We refer the reader to [1] and [2] for a select literature on Kirchhoff equations. Boundary value problems involving reflection of the argument, \( f(x, u(x), u(-x)) \), appear in some difference equations and were considered systematically by several authors. Our work is motivated by the results of [3], we find assumptions on the functions \( f \) and \( M \) in order to assure the iterative sequence
\[ u^{k+1}(x) = \frac{1}{2L} \int_{-L}^{L} G(x, s) f(s, u^k(s), u^k(-s)) \, ds, \quad (4) \]
where \( G \) is the Green’s function
\[ G(x, s) = \begin{cases} 
(s + L)(x - L) & \text{if } -L \leq s < x \leq L, \\
(x + L)(s - L) & \text{if } -L \leq x \leq s \leq L,
\end{cases} \quad (5) \]
converges to a solution of (1)-(2). We prove an existence theorem based on fixed point arguments and we discuss some numerical examples.

**Theorem 1.** Suppose some conditions on maximums of \( f, f', M, M' \) holds, then the problem (1)-(2) has a unique solution. In addition, this solution is the limit of the iterative sequence (4) with any initial approximation \( u^0 \) satisfying \( \|u^0\| \leq R \).

We compare our results with the ones given in [3], where the numerical solutions of the boundary value problem
\[ u'' - \alpha u' = f(x, u(x), u(-x)), \quad u(-1) = u(1) = 0, \quad \alpha \geq 0 \text{ is studied.} \]
Our simulations lead us to conclude that when \( \alpha = 0 \), our sequence is better than the Sharma’s sequence, specially if the expected solution \( u \) does not exhibit symmetries or anti-symmetries, like \( u = 1 - x^2 \) or \( u = x - x^3 \). On the other hand, if \( \alpha > 0 \) is small, then both schemes are comparable. But if \( \alpha > 0 \) is large, then our iterative scheme does not work.

**References**

