S-Convex Fuzzy Processes

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ABSTRACT

In 1967, Rockafellar [1] introduced the notion of convex processes (see also [2]). These are set-valued maps whose graphs are closed convex cones. For instance, they can be seen as the set-valued version of a continuous linear operator. Derivatives of some set-valued maps are closed convex processes, which is a desirable property for a derivative (see [3]). An important property of convex processes is that it is possible to transpose closed convex processes and use the benefits of duality theory. And as it is well known, these facts are very useful in optimization theory (see for instance [4], [5], [6], [7], [8]).

The extension of this notion to the fuzzy framework was done by Matloka [9]. Recently, Syan, Low and Wu [10] observed that Matloka definition is very strict. Therefore, they give another definition that extend the Matloka definition. In 2000 was introduced by the authors the concept M-convex fuzzy mapping [11], we observe that 1-convex fuzzy mapping is coincident with definition of convex process given in [10] (see Theorem 3.4, p. 195 in [10]) for the case m=1.

In 1978, Breckner introduced s-convex functions as a generalization of convex functions [12], and in 1993 studied the set-valued version [13]. We observe that convex processes are one particular case of s-convex set-valued maps. Also, in that one work Breckner proved the fact important that the set-valued map is s-convex if only if her support function is s-convex function. Other works related are [16], [17], [18].

In this work, we introduce the fuzzy version of the Breckner definition, and we will call this generalization s-convex fuzzy process. Moreover we will prove the equivalence with the s-convexity of the fuzzy support function and we study some properties.

The plan of the paper is as follows. In Section 2, we introduce the notations, definitions and preliminaries results used throughout the paper. In Section 3 we establish the main results and finally in Section 4 we show some algebraic properties and the connection with the fuzzy integral mean for fuzzy set-valued map.

Referências


