Modelling Hydrodynamic Interactions between a Mica Surface and a Mercury Drop

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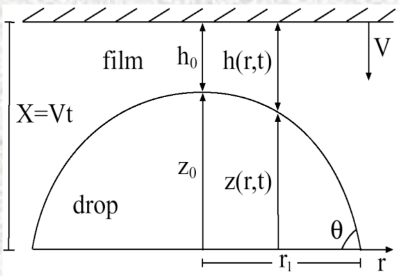
Abstract

A detailed experiment using the Surface Force Apparatus (SFA), in which a mica surface was driven towards a mercury drop was performed by Connor and Horn [Faraday Discuss. 2003, 123, 193-206]. Using video imaging they were able to obtain the time evolution of the profile of the mercury drop. Due to the relatively large size of the drops (radius about 2 mm) and velocity (about 23 μ m/s), the results show the presence of dimpling in the drop due to hydrodynamic pressure. The evolution of this dimple depends on surface potential that can be varied to provide strong repulsion to strong attraction.

We present a mathematical model, previously used to study deformation of much smaller drops, to predict the time evolution of the dimple including the main variables of the experiments. A feature of the model is the use of a new boundary condition, obtained by matched asymptotic expansions, to incorporate the weak deformation at the drop scale into the thin film scale. Numerical solutions show good agreement between theory and experiments for most of the tested cases.

Schematic of the Experiment



Governing Equations

The governing equations for the time evolution of the thickness of the thin film between a deformable drop and a mica surface, considering no-slip boundary conditions at the drop-water interface, are

$$\frac{\partial h}{\partial t} = \frac{1}{12\mu r}\frac{\partial}{\partial r}\left(rh^3\frac{\partial p}{\partial r}\right)$$

$$p + \Pi = \frac{2\sigma}{R_0} - \frac{\sigma}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right)$$

where h is the film thickness as a function of radial coordinate r and time t, p is the excess pressure, μ is the film viscosity, σ is the interfacial tension of the drop-film surface, R_0 is the undisturbed radius of the drop and Π is the disjoining pressure obtained from the Poisson- Boltzmann equation. The initial condition is given by

$$h(r,0) = h_0 + \frac{r^2}{2R_0}$$

and the boundary conditions, due to axial symmetry, are given by

$$\frac{\partial h}{\partial r} = 0 \text{ at } r = 0$$
$$\frac{\partial p}{\partial r} = 0 \text{ at } r = 0$$

At large radial distances

$$p \rightarrow 0 \text{ as } r \rightarrow \infty$$

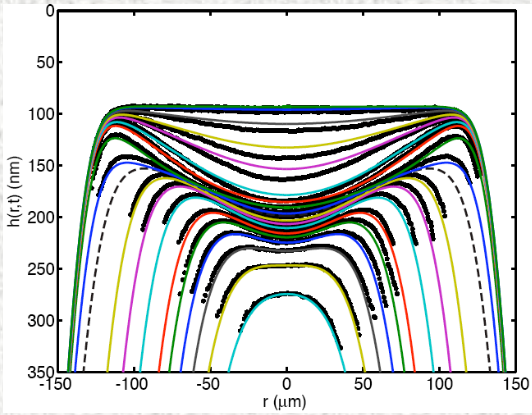
and the new boundary condition, developed in this work using asymptotic expansions, that takes into account deformation of the drop at large r is, considering that the three-phase contact line is pinned

$$\dot{h} + \dot{G}\left[1 + \frac{1}{2}\ln\left(\frac{r_{max}^2}{4R_0^2}\right) + \frac{1}{2}\ln\left(\frac{1+\cos\theta}{1-\cos\theta}\right)\right] = V$$

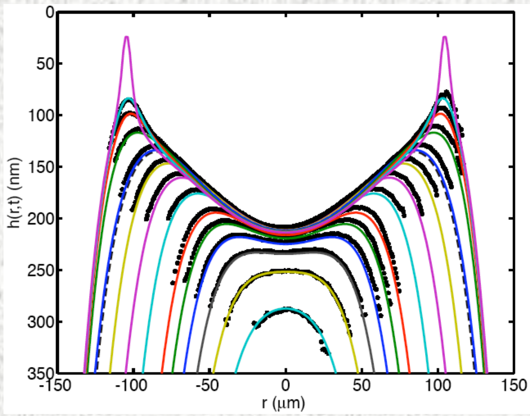
being V the approach velocity, θ the contact angle between drop and capillary.

Numerical Results compared with experiments

Presented results show the comparison between theory and experiment for 2 cases: repulsive and attractive. The parameters used were: V=24 μm/s, R_0=1.9 mm, σ=0.42 N/m, θ=52, ΔX=30 μm, and h_0=10 μm. For the repulsive case the solution is stable, being the final solution a flat surface in which the separation is governed by double layer forces. In the attractive case, as soon as the drop reaches a certain distance it jumps into the mica almost instantly due to strong short range attraction forces. The comparison is impressive in time and space, being the model able to predict the jump in time accurately.



Strong repulsion between mercury and mica



Strong attraction between mercury and mica

Conclusions and Perspectives

- New boundary condition:
  - deformation of the drop at large *r*.
  - effect of three-phase contact line.
- Incorporation of velocity dependent effects;
- Impressive agreement between theory and experiment;
- As a main perspective: solve the free drops interaction;
- Obtain analytical expressions for simplified cases;
- Study slip boundary condition and Marangoni effects in the model;
- Explain the the mechanisms of interaction between droplets with implications in emulsion stability, flotation collection, …