Diffusion Scheme for Noise Reduction Using Cellular Automata

Leandro S. Marturelli, Gilson A. Giraldi
LNCC–National Laboratory for Scientific Computing
Av. Getulio Vargas, 333, 25651-070 Petrópolis, RJ - Brazil
{schaefer,gilson}@lncc.br

Abstract

Cellular automata is a methodology that uses a discrete space to represents the state of each element of a domain, and this states can be changed according to a transition rule. In this work we use cellular automata to represent a digital image and to develop new diffusion scheme in a way to reduce the noise of the image. Then, the processed image will be converted in a binary one to perform segmentation. The main contribution is the fact that the proposed diffusion method works well in images with high noise intensity and the computational cost is small. We apply the methodology for medical images segmentations.

1. Introduction

Image segmentation is a technique very used in photo editing and medical image analysis. However, fully automation of these techniques is not trivial way due to many problems found in an image, such as noise and in homogeneities in the intensity field. Medical images have their own properties. In many cases they are gray level and the objects that should be segmented are different in their structure and appearance from the objects in photo editing. Besides, medical images carry, in most of times, a significant quantity of noise that perturbs the methods decreasing the efficiency and precision, leaving the representation of the edges not the desirable one. There are three segmentation methods, among many others, that are very useful: Region Growing, Threshold and Active Contours [1,2]. The first one works based in an initial value and join the neighbor that has the same intensity value; the second one is used to create a binary image where a decision is based in a given threshold; and the last one works deforming an initial curve over an image. Some segmentations methods use a diffusion scheme that can be formulated through Partial Differential Equations (PDE's) [3,4]. A remarkable work in this area came from the observation that the Gaussian filtering can be seen as the fundamental solution of the (linear) heat equation [5]. Then, Perona and Malik consider a non-linear heat equation and proposed their anisotropic (nonlinear) diffusion method [6]. Since then, PDE approaches have been used in multiscale techniques [7], image restoration, noise reduction and feature extraction [8]. In this work we use a diffusion scheme too, but not using a heat-equation neither PDE's. We propose a new diffusion scheme that is based in an idea of neighborhood equilibrium. The scheme considers some neighbors to achieve equilibrium of its intensities demanding a more suave difference between them or, in the best case, no difference. As consequence of it, considering that noise is a very small object, when the equilibrium is achieved the noise is blended with the background. For a representative object we must be careful with its edges. It's clear that the equilibrium will happen between the edges of the objects and the background, leaving the edges less representative. However, this softness is not restrictive and we will explain this behavior in section 3. So, this scheme can be view as a noise filter and can be used to pre-process an image and then apply a segmentation method. To evolve the process along discrete time and space, the cellular automata method is used to. This work is organized as follows: Section 2 presents the cellular automata method, Section 3 explain how the diffusion scheme works, in Section 4 we presents experimental results and Section 5 summarizes the conclusion and future works.

2. Related Works

Diffusion systems are an important class to investigate nonlinear behavior and they can be simulated by numerical techniques, such as finite difference methods or using an alternative approaches such as cellular automata. Many cellular automata for diffusion systems are constructed in such a way that they correspond qualitatively to the solutions of partial differential equations [10]. Besides, cellular automata are used, in image processing applications, to edge recognition [15], image segmentation using interactive multi-label [13] and composer for binary images with reaction-diffusion [14] among others ones. Up to our knowledge there is no use of cellular automata to evolve diffusion methods in order to segment digital images, which is propose in the next section.

Segmentation methods can take advantage of diffusion schemes in a two-step procedure. Firstly, diffusion is applied to smooth the digital image in order to solve local variations of the intensity field. Then, a segmentation method is applied with the processed image as the input.

There are many works for segmentation reported in the literature. Most of them are based on the following techniques:

Thresholding: The threshold segmentation separates the image elements (pixels) based only on their intensity. The success of this approach depends on the successful selection of a threshold. The thresholding methods have many variations: global (single threshold) or local threshold (depending on the position in the image), multi-thresholding, adaptive thresholding, etc. Generally, threshold-based methods are not suitable for segmenting textured images.

Clustering Techniques: Clustering refers to the classification of objects into groups according to certain properties of these objects. A standard procedure for clustering is to assign each pixel to the class of the nearest cluster mean. There is a variety of clustering algorithms in the literature, the most popular and simple being the K-means technique.

Region Growing Techniques: To produce spatially connected regions requires a criterion of geometrical proximity, in addition to the homogeneity criterion. A simple approach to image segmentation is to start with some user-defined pixels (seeds) representing distinct image regions and to grow them until the whole image is covered. Most region growing algorithms are based on splitting and merging techniques.

Active contours: Deformable models are called active contours or snakes in two dimensions. A deformable curve behaves like an elastic line. Initially, it is placed close to an object's boundary and it changes shape to match the shape of the boundary. The forces, which change the shape, are the external (or image) forces controlled by the image attributes and the internal forces, which control the curve regularity.

Bayesian Methods: They use probability calculus to quantify the plausibility of a hypothesis. In the case of image segmentation, this hypothesis is about the existence of a particular "hidden field" along with the data. A priori knowledge, which can be exploited to improve the results, is used to regularize the inference of the hidden field, given the data. Formal optimization techniques are then used to work on the posterior inference.

3. Diffusion Scheme

The diffusion method that we propose is based in a fact that if exist a neighbor with low intensity, compared with the central element, then this neighbor will earn a contribution from the central element to add with its own intensity. Besides, if the central element gave a contribution to a neighbor, then its own intensity must decrease in order to achieve the equilibrium condition. Figure 1 below show the idea in one dimension:

0	0	1	0	0

(a)

0	0.33	0.33	0.33	0

(b)

Figure 1: (a) Initial configuration; (b) After the contributions of the central element.

The new value for each element can be calculated by the expression:

$$u_i^{t+1} = \frac{u_i^t + u_{i+1}^t + u_{i-1}^t}{3} \tag{1}$$

where $\boldsymbol{i}$ is the central element.

If we consider an inverse case, where the central element is lower than the neighborhood, the effect of equilibrium will happen too (figure 2).

1	1	0	1	1

(a)

1	0.66	0.66	0.66	1

(b)

Figure 2: (a) Inverse initial configuration; (b) After the contributions to the central element.

Considering that elements with value one represents an object and elements with value zero represents the background, the direct (figure 1) and inverse (figure 2) cases will works in different way: The first one will, along the time, merge the object with the background; the second one will merge different objects depending the distance between them and the quantity of iterations.

Using the direct case, we will analyze an object with more joined elements (figure 3).

0	1	1	1	1	1	0

(a)

0.33	0.66	1	1	1	0.66	0.33

(b)

Figure 3: (a) Initial configuration; (b) After the contributions of the central element.

If the central element has neighbors with value equal to its own value, the value for the central element will not change, as shown below:

$$\begin{aligned} u_i^t &= u_{i+1}^t = u_{i-1}^t \\ u_i^{t+1} &= \frac{u_i^t + u_i^t + u_i^t}{3} = \frac{3u_i^t}{3} = u_i^t \end{aligned} \tag{2}$$

This fact agrees with the equilibrium condition. As consequence of Expression 2, the objects with joined no null elements will have only its edges merged with the background, preserving the interior of the objects.

Now, if we consider the small objects as a noise it will be merged with the background faster than bigger objects, implying in a noise reduction.

A more suitable scheme is shown in Figure 4, where we can see a 1D image with 400 pixels represented by a graphic (element x intensity). Figure 4.a represents the configuration of first iteration and Figure 4.b shows the configuration after 55 iterations. The three peaks represents three different objects with different sizes each one. The first one (on the left) has 10 pixels of length, the second has 50 pixels and the last one 150 pixels. After 55 iterations the Figure 4.b shows a significant decay only of the first object, which can be seem as a noise.

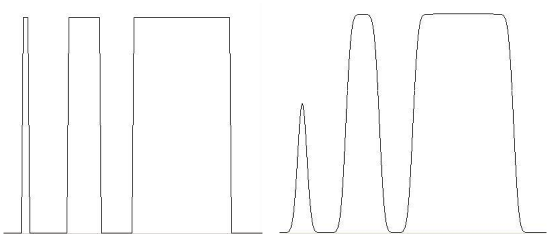

(a) (b)

Figure 4: (a) Initial configuration for 1D image with 400 pixels; (b) After 55 iterations.

Our interest is to segment the objects, but the bigger objects will be its edges decreased too, leaving the objects without the same initial shape. To by-pass this problem, we will transform the original image into a binary image, using a threshold $T \in [\min(I), \max(I)]$ to determine the value of the pixels, as shown in Expression 3 below:

$$\begin{aligned} &if\ (I_i < T) \\ &then\ I_i = 0 \\ &else\ I_i = 1 \end{aligned} \qquad (3)$$

where I_i is the intensity of the element *i*.

Following the example of the figure 3, if we choose T = 0.66 the binary image will represents exactly the shape of the objects in the original image and consequently bypassing the problem described above. So, choosing appropriately threshold, the binary image will has a good representation of the objects.

Now, using a binary image is very easy to find the contour of the desirable objects. We choose for our experiments the gradient filter:

1. *Compute the gradient of the elements*;
2. *If gradient is different of zero then this element is one point of the contour*.

The same analysis can be shown for two dimensions, which is a case for digital images.

4. Cellular Automata

In this section we will describe the utilization of the cellular automata in this work.

Von Neumann and Ulam introduced cellular automata as simple models in which to study biological processes such as self-reproduction [12]. Any system with many identical discrete elements undergoing deterministic local interactions may be modeled as cellular automata.

The cellular automata work defining a discrete grid where each element (or site) has a finite state. The sites can have its states changed according with a transition rule, which is defined for each case. Using a discrete time representation, these sites can be updated for each time step until achieve a stop condition.

In our context, a rectangular and regular grid is used to represent a digital image and each cell represents one pixels of the image. So, the initial configuration (t = 0) is exactly the original image. The transition rule used to update the state of the cells is in according with Expression 1, but now expanded for two dimensions:

$$u_{i,j}^{t+1} = \frac{u_{i,j}^{t} + u_{i+1,j}^{t} + u_{i-1,j}^{t} + u_{i,j+1}^{t} + u_{i,j-1}^{t}}{5} \qquad (4)$$

To determine which neighbors will be used in the diffusion scheme, we choose the Von Neumann neighborhood [11] picture in figure 5.

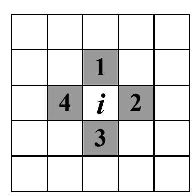


Figure 5: The Von Neumann neighborhood.

5. Experimental Results

In this section we will show the segmentation's results of seven different images. The first image we have added gaussian noise and then we verified the number of iterations needed to get the desirable contour. The image picture in figure 6 is representing the “U” shape in a homogenous background and added of noises. Figure 7 shows the evolution of diffusion scheme, figure 9 and 10 shows the analyses of increase the number of iterations and figure 11 shows five segmented images of cells.

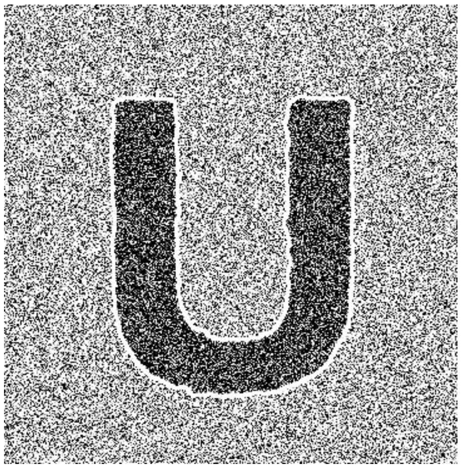
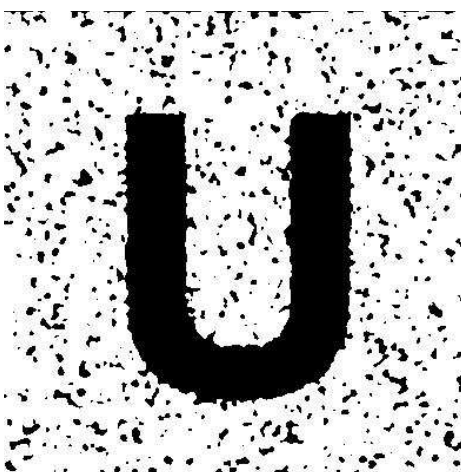
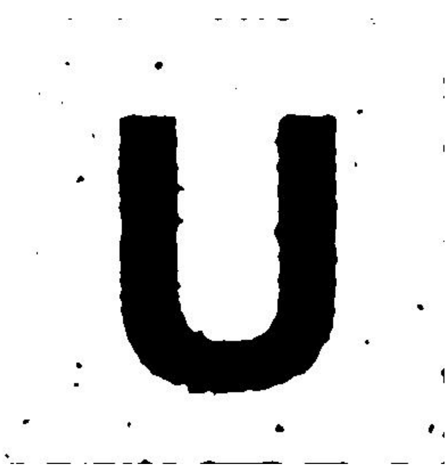


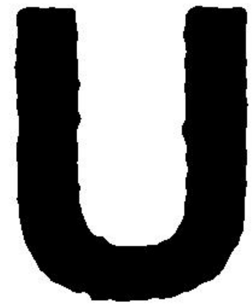
Figure 6: Image of “U” shape with gaussian noise



(a)



(b)



(c)

Figure 7: (a) Binary image of figure 6 for iteration 10; (b) Iteration 20; (c) Iteration 30.

For the image picture in figure 6 was needed 30 iterations to get an acceptable contour of the “U” shape, using a threshold $T = 0.7$.

To show the evolution of the diffusion scheme, figure 7 shows the binary images for picture 6 for iterations 10, 20 and 30, where we can see a gradual noise reduction. Continuing the analyses in the same image, we increase the number of iterations up to 150 and we get a more suave contour of “U” shape (figure 8).

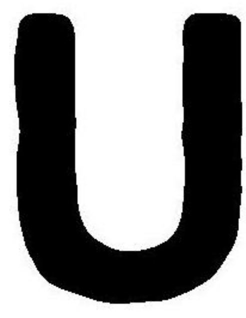
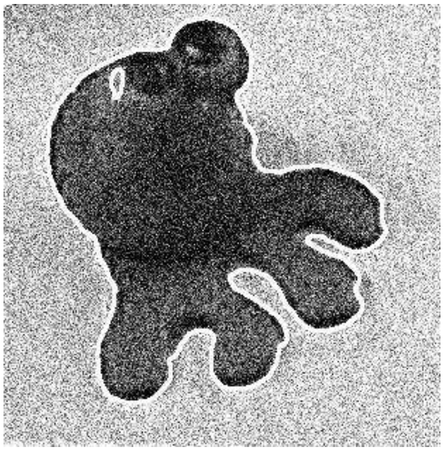
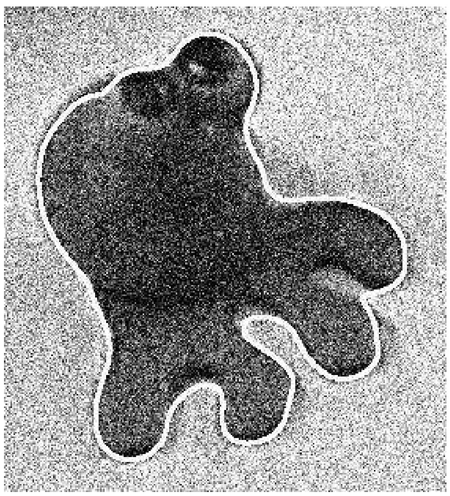


Figure 8: Binary image of figure 6 for iteration 150.

However, we did the same analyses to image picture in figure 9. We observed that when the number of iterations gets larger (figure 9.b), the contour of the object is not the desirable one. This fact is due to shape of objects. More diffusion implies in more equilibrium between neighbors and when objects, or parts of objects, are too near they can merge. Sometimes, choosing another threshold, the desirable contour can be found again, but this is not guaranteed.



(a)



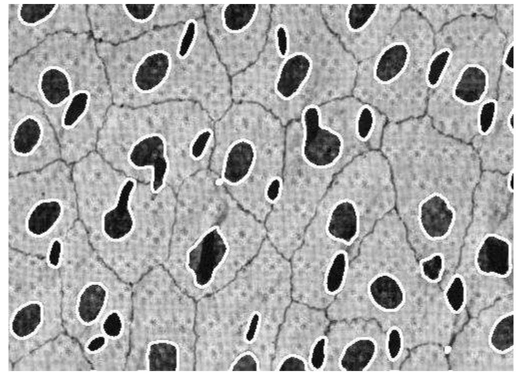
(b)

Figure 9: (a) Image segmented using $T = 0.55$ with 22 iterations; (b) After 150 iterations.

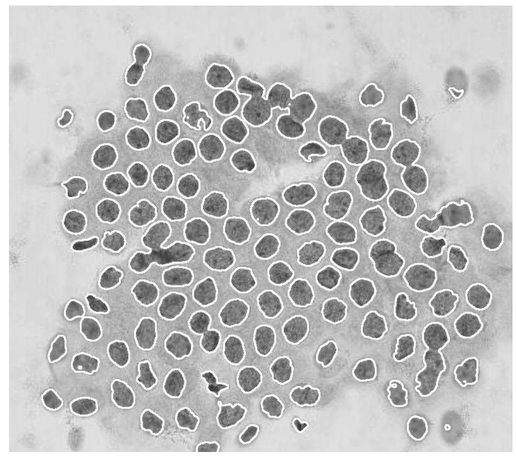
In table 1 below, we show the threshold (T) and number of iterations (NI) for the segmented images pictures in figure 11 (*a,b,c,d* and *e*). The spatial resolutions are respectively 505x366, 797x704, 540x327, 540x327 and 535x554.

Image	(a)	(b)	(c)	(d)	(e)
T	0.5	0.65	0.55	0.8	0.4
NI	20	15	40	130	100

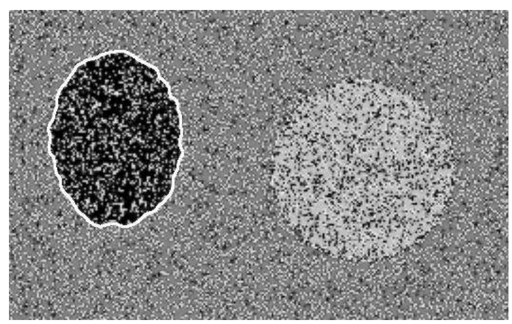
Table 1: Threshold and number of iterations for the images in figure 11.



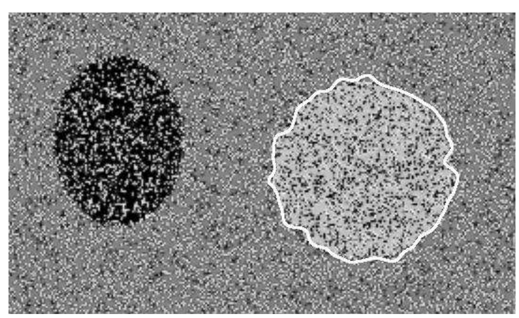
(a)



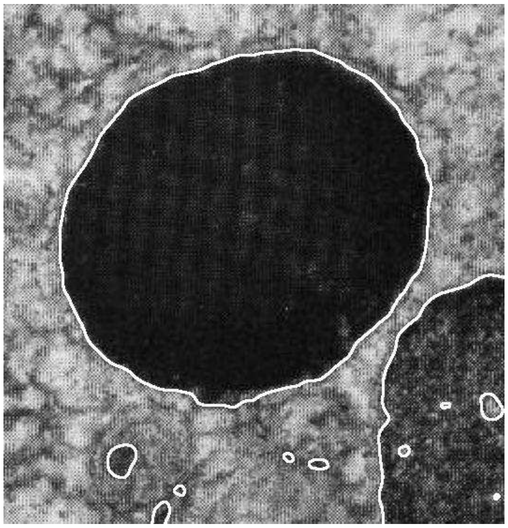
(b)



(c)



(d)



(e)

Figure 11: Some segmented images of cells.

6. Conclusions and Future Works

In this work we propose a utilization of a new noise reduction filter, which is based in a diffusion of image's pixels, used to pre-process images in order to segment them. We made use of a threshold to create a digital image and then to find contours of the objects using a gradient of the binary images. The methodology works robustly when submitted a variation of noise in the same image, only need more iterations when the noise were increased. We verified that more diffusion implies in more softness of the contours and the use of it depends on the shape of the objects.

For future works, we intend to create a condition to determine the quantity of iterations are need to filter an image without leave undesirable noises and without endanger the shape of objects of interest.

7. References

[1] T. McInerney and D. Terzopoulos. Deformable models in medical images analysis: a survey. Medical Image Analysis, 1(2):91--108, 1996. http://citeseer.ist.psu.edu/mcinerney96deformable.html

[2] R. Pohle, K. D. Toennies, Segmentation of medical images using adaptive region growing. Proc. SPIE (Medical Imaging 2001), 4322, to appear.

[3] C. Xu, D. L. Pham, and J. L. Prince, "Medical Image Segmentation Using Deformable Models," *SPIE Handbook on Medical Imaging* -- Volume III: Medical Image Analysis, edited by J.M. Fitzpatrick and M. Sonka, May 2000.

[4] C. Xu and J. L. Prince, ``Snakes, Shapes, and Gradient Vector Flow," *IEEE Transactions on Image Processing*, 7(3), pp. 359-369, March 1998.

[5] J. Koenderink. The structure of images. *Biol. Cybern.*, 50:363–370, 1984.

[6] P. Perona and J. Malik. Scale-space and edge detection using anisotropic diffusion. *IEEE Trans. on Patter Analusis and Mach. Intell.*, 12(7):629–639, July 1990.

[7] L.M. Florack, A.H. Salden, B.M. Ter Haar Romery, J.J Koenderink, and M.A. Viergever. Nonlinear scalespace. *Image and Vision Computing*, 13(4):279–294, May 1995.

[8] Special issue on partial differential equations and geometry-driven diffusion in image processing and analysis. *IEEE Trans. Image Proc.*, March 1998.

[9] http://bme.med.upatras.gr/improc

[10] Jorg R. Weimar. Cellular automata for reaction-diffusion systems. Parallel Computing, 23(11):1699--1715, 1997.

[11]http://mathworld.wolfram.com/vonNeumannNeighborhood.html

[12] J. von Neumann, ``The General and Logical Theory of Automata", in J. von Neumann. ``Collected Works" (ed. A.H. Taub), Vol. 5, p.288; J. von Neumann, ``Theory of Self-Reproducing Automata", (ed. A.W. Burks), Univ. of Illinois Press (1966); ed. A.W. Burks, ``Essays on Cellular Automata", Univ. of Illinois Press (1970).

[13] Vezhnevets V., Konouchine V. ""GrowCut" - Interactive Multi-Label N-D Image Segmentation By Cellular Automata". Accepted to Graphicon-2005, Moscow, Russia, 2005.

[14] Asai T., Ikebe M., Hirose T., and Amemiya Y., "A quadrilateral-object composer for binary images with reaction-diffusion cellular automata," *International Journal of Parallel, Emergent and Distributed Systems*, vol. 20, no. 1, pp. 57-68, (2005).

[15]Cellular Automata Modeling in Edge Recognition. Chen Yang Hao Ye, G. Z. Wang. Dept. of Automation, Tsinghua Uinv. Dept. of Automation, Tsinghua Uinv. Beijing, 100084, P .R.