Vector-spinor duality and singular solutions of the Seiberg-Witten equations in $\mathbb{R}^3$

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Given a physical system defined on a configuration space M, there are various instances where it is useful to employ (extensions of) fibrations $P \to M$ to lift the corresponding equations of motion from M to P. For example, the natural extension of the Hopf fibration $S^3 \to S^2$ to $\mathbb{R}^4 \to \mathbb{R}^3$ (defining the so-called Kustaanheimo-Stiefel transformation) can be used to map the Kepler problem in $\mathbb{R}^3$ to a harmonic oscillator problem in $\mathbb{R}^4$. This construction has been recurrently used to regularize and effectively calculate orbits of celestial objects, besides giving rise to various applications in atomic physics. In this work, we apply this idea to the case when M, instead of representing the configuration space of a particle, is the target space of a given field theory. Specifically, we show that by lifting the equations of magnetostatics (in the sense above), it is possible to obtain the Seiberg-Witten equations (SWE) on $\mathbb{R}^3$ provided that a certain constraint is imposed on the resulting fields. Moreover, we show that this procedure naturally gives rise to an iterative method to generate particular solutions to the SWE and Freund equations on $\mathbb{R}^3$. As a result, some known solutions are recovered and a new (to the best of our knowledge) axisymmetric singular solution to the SWE on $\mathbb{R}^3$ is obtained.

Keywords: Seiberg-Witten equations, Hopf map, Singular solutions to PDEs, Kustaanheimo-Stiefel transformation.

References

[1] P. Kustaanheimo and E. Stiefel, "Perturbation theory of Kepler motion based on spinor regularization", J. fur die Reine und Angew. Math. **218**, 204 (1965).

[2] M. Kibler, "Application of non-bijective transformations to various potentials", in *Group Theoretical Methods in Physics*, H. D. Doebner, J. D. Hennig and T.D. Palev (eds.), Springer-Verlag, Berlin, 1988: Lecture Notes in Physics **313**, 238 (1988).

[3] T. Bartsch, "The Kustaanheimo–Stiefel transformation in geometric algebra", J. Phys. A: Math. Gen. **36** 6963–6978 (2003), physics/0301017.

[4] Nergiz S. and Saçlıoğlu C., "Liouville vortex and φ^4 kink solutions of the Seiberg-Witten equations", J. Math. Phys. **37**, 3753–3759 (1996), hep-th/9602088.

[5] Adam C., Muratori B. and Nash C., "On non-L^2 solutions to the Seiberg-Witten equations", J. Math. Phys. **41**, 5875-5882 (2000), hep-th/0003125.