A New Class of Variable Metric Interior-Proximal Method for the Variational Inequality Problem

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Let C be a polyhedral set on $I\!R^m$ defined by

$$C := \{x \in I\!R^m : Ax \leq b\},$$

where $A \in I\!R^{p\times m}$, $b \in I\!R^p$, $p \geq m$. Let T be a maximal monotone set valued map. We suppose that the matrix A has full rank, that is, $\text{rank}A = m$, $\text{int}\, C = \{x : Ax < b\}$ is nonempty and

$$\text{dom}\, T \cap \text{int}\, C \neq \emptyset, \tag{1}$$

where dom T is the effective domain, that is, $\text{dom}\, T = \{x : T(x) \neq \emptyset\}$.
We consider the variational inequality problem $(VI(T,C))$:
Find $x^* \in C$ and $g^* \in T(x^*)$ satisfying

$$\langle g^*,\ x - x^*\rangle \geq 0,\ \forall x \in C\text{''}.$$

We suppose that $\mathcal{S}$, the solution set of $(VI(TC))$, is nonempty.
It is easy to verify that when $T(x) = \partial f(x)$ with f lower semicontinuous proper convex function, the problem $(VI(T,C))$ becomes $\min\ f(x)$ such that $x \in C$.
A well known method for solving $(VI(T,C))$ is the proximal algorithm, see for instance, [6]. Given $\lambda_k \geq \lambda > 0$, the inexact version of the proximal point algorithm in [6] generates iteratively sequences $\{x^k\} \subset C$, $\{e^k\} \subset I\!R^m$ satisfying:

$$g^k + \lambda_k^{-1}(x^k - x^{k-1}) = e^k, \qquad g^k \in \hat{T}(x^k),$$

where $\hat{T}(\cdot) = T(\cdot) + N_C(\cdot)$, N_C denotes the normal cone and e^k the error sequence.
Many papers have concentrated on generalization of the proximal point algorithm replacing the linear term $x^k - x^{k-1}$, by some nonlinear functionals $r(x^k, x^{k-1})$ based on entropic proximal terms arising from appropriately formulated Bregman functions, see for example [3] and [4], or entropic ϕ-divergence, see for example [7], and leading to interior point proximal methods for variational inequality problems.
For solving the problem $(VI(T,C))$, we propose a family of inexact interior proximal methods which the kernels are metrics generated by general diagonal matrices, constructed upon the r-power of the iterates, with $r \geq 0$; moreover, in these methods the regularization parameter is conveniently chosen at each iteration. We obtain the well-definedness of the sequence generated by proposed algorithm and we prove under mild assumptions global convergence to a solution. We also get a particular case of the method to solve minimization under positivity constraints.

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References

[1] Auslender, A., Teboulle, M. and Ben-Tiba, S., A Logarithmic-Quadratic Proximal Method for Variational Inequalities, *Computational Optimization and Applications* 12, 31-40, (1999).

[2] Auslender, A., Teboulle, M. and Ben-Tiba, S, Interior Proximal and Multiplier Methods Based on Second Order Homogeneous Kernels. *Math. of Operations Research*, 24, 645-688, (1999).

[3] Burachik, R.S. and Iusem, A.N., A generalized proximal point algorithm for the variational inequality problem in a Hilbert Space, *SIAM Journal on Optimization*, 8, 197-216, (1998).

[4] Eckstein, J., Aproximate iterations in Bregman-function-based proximal algorithms,

Mathematical Programming, 83, 113-123, (1998).

[5] Oliveira, G.L., Oliveira, P.R., A New Class of Proximal Interior-Point Methods for Optimization under Positivity Constraints, ES-570, February 2002, PESC/COPPE-UFRJ.

[6] Rockafellar, R.T., Monotone Operators and the Proximal Point Algorithm, *SIAM J. Of Control and Optimization*, 14, 877-898, (1976).

[7] Teboulle, M. Convergence of proximal-like algorithms, *SIAM J. of Optimization*, 7, 1069-1083, (1997).