Stability of linear time-varying systems through quadratically parameter-dependent Lyapunov functions

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Abstract

This paper aims on the stability analysis of linear continuous-time systems whose dynamic matrices are affected by one uncertain time-varying parameter, assumed as bounded, continuously differentiable, with bounded rates of variation. A sufficient condition to obtain a quadratically parameter-dependent Lyapunov function is given in terms of linear matrix inequalities whose feasibility assures that the origin is a globally asymptotically stable equilibrium point for the system. Numerical results illustrate the efficiency of the proposed condition, which provides less conservative estimates of stability domains than similar tests in the literature.

Key words: Linear time-varying systems; Polytopic uncertainty; Parameter-dependent Lyapunov functions; Linear matrix inequalities.

1 Introduction

The analysis of stability of linear time-varying systems is an important topic in systems theory [11]. It is well known that the asymptotic stability of a time-varying system cannot be based on eigenvalue calculation, except in very few special cases, as for instance for slowly time-varying systems. The search for Lyapunov functions that can provide a fair evaluation of stability domains for linear time-varying systems is still a matter of studies and a particularly useful approach is the one in which the Lyapunov function can be obtained from the solution of linear matrix inequalities (LMIs) [4].

Continuous-time linear systems whose dynamic matrices are affected by bounded uncertain time-varying parameters have been systematically investigated through sufficient LMI stability conditions, as for instance in [6], [9], where the uncertainties admit a linear fractional transformation representation, or in [7], [13], where the system is written with affine dependence on the uncertain parameters, and in [5], [8], [12], for polytopic uncertainties. In the case of unbounded rates of variation of the uncertain parameters, the use of a Lyapunov function with a common matrix (quadratic stability, [3]) has been important to investigate stability, and also for robust filter and control design [4]. Less conservative results can be obtained using more complex Lyapunov functions, as the piecewise quadratic [1], [10], which encompass the results from common quadratic Lyapunov functions, but at the price of solving LMI conditions associated with searches for a set of parameters in unbounded spaces, leading to high computational burdens. However, the assumption of arbitrary rates of parametric variation may lead to conservative evaluations when the parameters of the system have bounded time-derivatives. In this case, parameter-dependent Lyapunov functions are particularly useful since the bounds on the rates of parametric variation can be used as an information in the LMI conditions which test the system stability, as for instance in [2], [7], [12], [13]. The conditions given in [7] and in [12] are based on Lyapunov functions with affine dependence on the uncertain time-varying parameters. The conditions given in [13] explore the use of Lyapunov functions that are quadratically dependent on the uncertain parameters, leading to less conservative results but the required computational effort is usually high and depends on the choice of a factorization for the original uncertain system.

This paper is focused on the stability analysis of linear systems whose dynamic matrices are affected by one uncertain time-varying parameter which is bounded, continuously differentiable and has bounded rates of variation. Sufficient LMI conditions are given to solve the following two problems: i) given the lower and upper bounds of the uncertain parameter and of its time-derivative, verify whether the origin is a globally asymptotically equilibrium point of the system; ii) when the bounds on the rates of parametric variation are not given *a priori*, search for the maximum bounds of the time-derivatives for which the origin is a globally asymptotically equilibrium point. Problem i) is addressed through the feasibility of LMI conditions and problem ii) is addressed using a unidimensional search method associated with the feasibility of LMI conditions. The tests proposed here rely on Lyapunov functions that are quadratic on the state and quadratically dependent on the uncertain parameter, yielding better results than those from conditions based on affine Lyapunov functions from [7] and [12] and also than those from the conditions given in [13], as shown by numerical examples.

2 Problem formulation

Consider the system

$$\dot{x}(t) = \big(A_0 + \theta(t)A_\theta\big)x(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ for all t belonging to the set of nonnegative real numbers $\mathbb{R}^+$, $A_0 \in \mathbb{R}^{n\times n}$, $A_\theta \in \mathbb{R}^{n\times n}$. The parameter $\theta(t)$ is assumed to be uncertain and time-varying, lying for all $t \in \mathbb{R}^+$ in the real interval $[\underline{\theta}, \overline{\theta}]$ with no discontinuity and subject to bounded rates of variation $|\ \dot{\theta}(t)\ | \leq \nu$, for all $t \in \mathbb{R}^+$.

The main objective of the paper is the following: for a given set of parameters $\underline{\theta}$, $\overline{\theta}$, ν, determine if the origin $x = \mathbf{0}$ is a globally asymptotically stable equilibrium point of system (1).

System (1) can be represented by an equivalent polytopic system given by

$$\dot{x}(t) = A(\alpha(t))x(t) \tag{2}$$

where $A(\alpha(t)) \in \mathbb{R}^{n\times n}$ is defined as

$$A(\alpha(t)) = \alpha_1(t)A_1 + \alpha_2(t)A_2, \quad \alpha_i(t) \geq 0, \quad i = 1, 2, \quad \alpha_1(t) + \alpha_2(t) = 1 \tag{3}$$

Matrices A_1 and A_2 are the vertices of the polytope of matrices (3) given by

$$A_1 = A_0 + \underline{\theta}A_\theta, \quad A_2 = A_0 + \overline{\theta}A_\theta \tag{4}$$

Notice that from $A_0 + \theta(t)A_\theta = \alpha_1(t)A_1 + \alpha_2(t)A_2$, with A_1 and A_2 given by (4), one has

$$\theta(t) = \alpha_1(t)\underline{\theta} + \alpha_2(t)\overline{\theta} \tag{5}$$

From (3), $\alpha_2(t) = 1 - \alpha_1(t)$, which allows to rewrite (5) as

$$\theta(t) = \overline{\theta} + \alpha_1(t)(\underline{\theta} - \overline{\theta}) \tag{6}$$

leading to

$$|\ \dot{\theta}(t)\ | = |\ \dot{\alpha}_1(t)\ ||\ \underline{\theta} - \overline{\theta}\ | \tag{7}$$

From the specification $|\ \dot{\theta}\ | \leq \nu$, one has

$$|\ \dot{\alpha}_1(t)\ | \leq \frac{\nu}{|\ \underline{\theta} - \overline{\theta}\ |} \triangleq \rho_1 \tag{8}$$

Now, it is possible to recast the original problem in the following way: for a given set of parameters $\underline{\theta}$, $\overline{\theta}$, ν, find the equivalent polytopic system (2)-(3), with vertices A_1, A_2 given by (4) and with the bound on the rates of parametric variation ρ_1 given by (8). Then, determine if the origin $x = \mathbf{0}$ is a globally asymptotically stable equilibrium point of system of (2). This is sufficient to ensure the global asymptotic stability of the origin of system (1).

3 Preliminary results

Consider the parameter-dependent Lyapunov function[1]

$$v(x, \alpha) = x'P(\alpha)x\ , \ \ P(\alpha) = P(\alpha)' > 0 \tag{9}$$

If the time-derivative of function $v(x, \alpha)$, given by

$$\dot{v}(x, \alpha) = x'\Big(A(\alpha)'P(\alpha) + P(\alpha)A(\alpha) + \dot{P}(\alpha)\Big)x \tag{10}$$

is negative for all $x \neq \mathbf{0}$, that is, if

$$A(\alpha)'P(\alpha) + P(\alpha)A(\alpha) + \dot{P}(\alpha) < 0 \tag{11}$$

then the origin $x = \mathbf{0}$ is a globally asymptotically stable equilibrium point of system (2). Notice that $\dot{P}(\alpha)$ denotes the time-derivative of matrix $P(\alpha)$, given by

$$\dot{P}(\alpha) = \frac{\partial P(\alpha)}{\partial \alpha}\ \frac{d\alpha}{dt} \tag{12}$$

The following lemma reproduces the well known result of quadratic stability [4].

Lemma 1 *If there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ such that*

$$A_1'P + PA_1 < 0 \tag{13}$$

$$A_2'P + PA_2 < 0 \tag{14}$$

then the origin $x = \mathbf{0}$ is a globally asymptotically stable equilibrium point of system (2).

Proof: Thanks to convexity, conditions of Lemma 1 are sufficient to ensure (11), with $P(\alpha) = P$ and $A(\alpha)$ given by (3). ∎

The existence of a solution for Lemma 1 (quadratic stability) guarantees the system stability for any arbitrary rate of parametric variation. Although this test has a low numerical complexity, the results are usually conservative since one common matrix has to assure the stability of the entire domain of uncertainties.

A less conservative condition to evaluate the stability of time-varying systems subject to bounded rates of parametric variation is given by the next lemma [12].

[1] The time dependence of the variables will be omitted from this point to the end of section 4, for sake of simplicity.

Lemma 2 *For a given parameter $\rho_1 \in \mathbb{R}^+$, if there exist symmetric positive definite matrices $P_1 \in \mathbb{R}^{n\times n}$, $P_2 \in \mathbb{R}^{n\times n}$ such that*[2]

$$A_1'P_1 + P_1A_1 \pm \rho_1(P_1 - P_2) < 0 \quad (15)$$

$$A_2'P_2 + P_2A_2 \pm \rho_1(P_1 - P_2) < 0 \quad (16)$$

$$A_1'P_2 + P_2A_1 + A_2'P_1 + P_1A_2 \pm 2\rho_1(P_1 - P_2) < 0 \quad (17)$$

then the origin $x = \mathbf{0}$ is a globally asymptotically stable equilibrium point of system (2) for bounded rates of parametric variation (8).

Proof: Using

$$P(\alpha) = \alpha_1P_1 + \alpha_2P_2 \quad (18)$$

and $\dot\alpha_2 = -\dot\alpha_1$, one has

$$\dot P(\alpha) = \dot\alpha_1(P_1 - P_2) \quad (19)$$

Replacing $P(\alpha)$ by (18) and $A(\alpha)$ by (3) in (11), and replacing $\dot P(\alpha)$ by (19) multiplied by $(\alpha_1+\alpha_2)^2 = 1$, it is possible to rewrite the lefthand side of (11) as

$$\alpha_1^2\big(A_1'P_1 + P_1A_1 + \dot\alpha_1(P_1 - P_2)\big) + \alpha_2^2\big(A_2'P_2 + P_2A_2 + \dot\alpha_1(P_1 - P_2)\big) + \alpha_1\alpha_2\big(A_1'P_2+P_2A_1+A_2'P_1+P_1A_2+2\dot\alpha_1(P_1-P_2)\big) \quad (20)$$

Taking into account that $\mid \dot\alpha_1 \mid\leq \rho_1$, the feasibility of the conditions of Lemma 2 is sufficient to assure that each one of the terms of (20) is negative definite and thus (20) is negative definite for all $\alpha_1 \geq 0$, $\alpha_2 \geq 0$, $\alpha_1+\alpha_2 = 1$, ensuring the global asymptotic stability of the origin of system (2) for bounded rates of parametric variation (8). ∎

Notice that the bound on the time-derivatives of the parameters ρ_1 is taken into account by the conditions of Lemma 2. For instance, when $\rho_1 = 0$, conditions (15)-(17) assess the stability of the system for time-invariant uncertainties and when ρ_1 is arbitrarily high, it is possible to verify the stability for arbitrarily fast parameter variation.

Regarding to the relationship between the conditions of lemmas 1 and 2, observe that if (13)-(14) are feasible with a solution P, then $P_1 = P_2 = P$ is also a solution of (15)-(17) for any $\rho_1 \in \mathbb{R}^+$. On the other hand, if the conditions of Lemma 2 are feasible for ρ_1 arbitrarily high, then the solution tends to $P_1 = P_2 = P$, in order to eliminate the influence of the terms multiplied by $\pm\rho_1$ in (15)-(17), pointing out to the quadratic stability of the system for arbitrarily high values of ρ_1. Thus, the conditions of Lemma 2 always provide better (or equal, in the worst case) results than the quadratic stability of Lemma 1, as can be seen by numerical comparisons in [12], which also indicate that (15)-(17) yield less conservative estimations of stability domains than the conditions given in [7], based on affinely parameter-dependent Lyapunov functions.

[2] The symbol $\pm$ in the inequalities indicates that two LMIs have to be tested: one for $+\rho_1$ and other for $-\rho_1$.

4 Main result

Following the ideas used in Lemma 2, a new sufficient condition based on quadratically parameter-dependent Lyapunov functions is stated in the next theorem.

Theorem 1 *For a given parameter $\rho_1 \in \mathbb{R}^+$, if there exist symmetric positive definite matrices $P_1 \in \mathbb{R}^{n\times n}$, $P_{12} \in \mathbb{R}^{n\times n}$, $P_2 \in \mathbb{R}^{n\times n}$ such that*

$$A_1'P_1 + P_1A_1 \pm \rho_1(2P_1 - P_{12}) < 0 \quad (21)$$

$$A_1'P_{12} + P_{12}A_1 + A_2'P_1 + P_1A_2 \pm \rho_1(4P_1 - P_{12} - 2P_2) < 0 \quad (22)$$

$$A_1'P_2 + P_2A_1 + A_2'P_{12} + P_{12}A_2 \pm \rho_1(2P_1 + P_{12} - 4P_2) < 0 \quad (23)$$

$$A_2'P_2 + P_2A_2 \pm \rho_1(P_{12} - 2P_2) < 0 \quad (24)$$

then the origin $x = \mathbf{0}$ is a globally asymptotically stable equilibrium point of system (2) for bounded rates of parametric variation (8).

Proof: Consider the quadratically parameter-dependent Lyapunov function

$$P(\alpha) = {\alpha_1}^2P_1 + \alpha_1\alpha_2P_{12} + {\alpha_2}^2P_2 \quad (25)$$

with

$$P_1 = P_1' > 0, \quad P_{12} = P_{12}' > 0, \quad P_2 = P_2' > 0 \quad (26)$$

The time-derivative of (25) is given by

$$\dot P(\alpha) = 2\alpha_1\dot\alpha_1P_1 + \dot\alpha_1\alpha_2P_{12} + \alpha_1\dot\alpha_2P_{12} + 2\alpha_2\dot\alpha_2P_2 \quad (27)$$

and can be expressed as

$$\dot P(\alpha) = 2\alpha_1\dot\alpha_1P_1 + \dot\alpha_1\alpha_2P_{12} - \alpha_1\dot\alpha_1P_{12} - 2\alpha_2\dot\alpha_1P_2 \quad (28)$$

Multiplying (28) by $(\alpha_1 + \alpha_2)^2 = 1$, one has

$$\dot P(\alpha) = \alpha_1^3(\dot\alpha_1(2P_1 - P_{12})) + \alpha_1^2\alpha_2(\dot\alpha_1(4P_1 - P_{12} - 2P_2)) + \alpha_1\alpha_2^2(\dot\alpha_1(2P_1+P_{12}-4P_2)) + \alpha_2^3(\dot\alpha_1(P_{12}-2P_2)) \quad (29)$$

Using (3), (25) and (29), the lefthand side of (11) can be rewritten as

$$\alpha_1^3\big(A_1'P_1 + P_1A_1 + \dot\alpha_1(2P_1 - P_{12})\big) + \alpha_1^2\alpha_2\big(A_1'P_{12} + P_{12}A_1 + A_2'P_1 + P_1A_2 + \dot\alpha_1(4P_1 - P_{12} - 2P_2)\big) + \alpha_1\alpha_2^2\big(A_1'P_2 + P_2A_1 + A_2'P_{12} + P_{12}A_2 + \dot\alpha_1(2P_1 + P_{12} - 4P_2)\big) + \alpha_2^3\big(A_2'P_2 + P_2A_2 + \dot\alpha_1(P_{12} - 2P_2)\big) \quad (30)$$

Since $|\ \dot\alpha_1\ | \leq \rho_1$, the feasibility of the conditions of Theorem 1 is sufficient to assure that each one of the terms of (30) is negative definite, implying that (30) is negative definite for all $\alpha_1 \geq 0$, $\alpha_2 \geq 0$, $\alpha_1 + \alpha_2 = 1$, thus ensuring the global asymptotic stability of the origin of system (2) for bounded rates of parametric variation (8). ∎

A first remark on the conditions of Theorem 1 is that more scalar variables are used in the tests (21)-(24) than in the tests of lemmas 1 and 2, which can provide less conservative evaluations of stability at the price of a slightly higher numerical complexity.

As in the case of Lemma 2, the conditions (21)-(24) can be used from $\rho_1 = 0$ to ρ_1 arbitrarily high. Particularly, if there exists a solution for ρ_1 arbitrarily high, it will tend to $P_1 = P_2 = P$, $P_{12} = 2P$, which eliminates the contribution of the terms multiplied by $\pm\rho_1$ in (21)-(24).

Notice that if the conditions of Lemma 2 are feasible, then P_1, P_2 and $P_{12} = P_1 + P_2$ yield a feasible solution to Theorem 1. Actually, in this case, the LMIs of Theorem 1 can be obtained as a linear combination of (15), (16) and (17).

The conditions of Theorem 1 can provide (with a lower computational burden) less conservative evaluations of stability than the conditions given in [13], also based on quadratically parameter-dependent Lyapunov functions, as shown in the sequel by numerical examples.

The number of scalar variables of conditions of (21)-(24) is $K_{T1} = 3n(n+1)/2$, and the number of LMI rows is $L_{T1} = 11n$ (including $P_1 > 0$, $P_{12} > 0$, $P_2 > 0$), where n is the order of the system. The numerical complexity depends on the LMI solver to be used. For instance, with the LMI Control Toolbox of Matlab (used in the numerical experiments in this paper), one has that the numerical complexity is proportional to K^3L, which means that the problem can be solved in polynomial time. Figure 1 shows a comparison of the numerical complexity of the LMI conditions of Lemma 2, Theorem 1 and the conditions from [7] and [13], all based on parameter-dependent Lyapunov functions. Observe that in the case of Lyapunov functions affinely dependent on the parameters, the conditions of Lemma 2 have lower numerical complexity than those from [7], and, in the case of Lyapunov functions quadratically dependent on the parameters, the conditions of Theorem 1 present lower numerical complexity than those from [13].

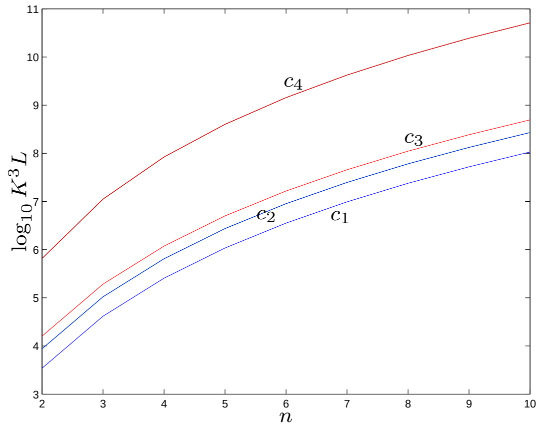

Figure 1: Numerical complexity using the LMI Control Toolbox of Matlab for the conditions from Lemma 2 (c_1), from [7] (c_2), from Theorem 1 (c_3) and from [13] (c_4) as a function of the order of the system n.

When the bounds on the rates of variation of the uncertain parameters are not given *a priori*, the conditions of Theorem 1 can be used to estimate the bounds of the stability domains of the system, as stated in the next corollary.

Corollary 1 *In the case $\rho_1 \in \mathbb{R}^+$ is not given a priori, use a unidimensional search method (for instance, the bisection method) to find the maximum value of ρ_1 for which the conditions of Theorem 1 are feasible.*

Notice that to maximize ρ_1 subject to (21)-(24) is a nonconvex optimization problem, but a simple unidimensional search on ρ_1 allows one to find the maximum value of ρ_1 such that the conditions of Theorem 1 hold.

5 Numerical examples

The next examples illustrate the efficiency of the conditions of Theorem 1 to estimate bounds of stability of linear systems affected by one uncertain bounded parameter with bounded rates of parametric variation.

Example 1 Consider the system borrowed from [13]

$$\dot x(t) = \left\{ \begin{bmatrix} 8 & -9 \\ 120 & -18 \end{bmatrix} + \theta(t) \begin{bmatrix} -108 & 9 \\ -120 & 17 \end{bmatrix} \right\} x(t) \quad (31)$$

where the uncertain time-varying parameter $\theta(t)$ lies in the interval $0 \leq \theta(t) \leq 1$, with rates of variation given by $|\ \dot\theta(t)\ | \leq \nu$. Notice that this system can be rewritten into the polytopic representation

(2)-(3), with vertices obtained from (4) given by

$$A_1 = \begin{bmatrix} 8 & -9 \\ 120 & -18 \end{bmatrix}, \text{ for } \theta(t) = \underline{\theta} = 0$$
$$A_2 = \begin{bmatrix} -100 & 0 \\ 0 & -1 \end{bmatrix}, \text{ for } \theta(t) = \overline{\theta} = 1 \quad (32)$$

and with $| \dot{\alpha}_1(t) |$ given by (8), leading to $\nu = \rho_1$.

The conditions given in [7] and the conditions of Lemma 2, both based on affinely parameter-dependent Lyapunov functions, provide the values $\nu = 62.50$ and $\nu = 63.25$, respectively. A better estimate for the maximum value of the bound on the rates of parametric variation given by $\nu = 66.81$, is obtained from the conditions given in [13], based on quadratically dependent Lyapunov functions. A high improvement on the estimate of the stability domain of this system is obtained using the conditions of Theorem 1 as indicated in Corollary 1, with the bisection method, which assures that the system is stable for $\nu = 99.6$, providing an enlargement of 49% on the bound of stability given by [13].

Example 2 As a second example, consider the system

$$\dot{x}(t) = \left\{ \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} + \theta(t) \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \right\} x(t) \quad (33)$$

where $0 \leq \theta(t) \leq \overline{\theta}$, which was studied in [14] in the context of arbitrary rates of parametric variations. Here, the objective is to assess the stability of (33) for bounded rates of parametric variation given by $| \dot{\theta}(t) | \leq \nu$. This system can be expressed in the polytopic representation (2)-(3), with vertices obtained from (4) given by

$$A_1 = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}, \text{ for } \theta(t) = 0$$
$$A_2 = \begin{bmatrix} 0 & 1 \\ -(2+\overline{\theta}) & -1 \end{bmatrix}, \text{ for } \theta(t) = \overline{\theta} \quad (34)$$

with $| \dot{\alpha}_1(t) |$ given by (8), which leads to $\nu = \overline{\theta}\rho_1$.

Figure 2 shows a comparison of the bounds of the domain of stability of system (33) obtained from the conditions given in [7], in [13], and from the conditions of Lemma 2 and Theorem 1, used as indicated in Corollary 1.

The curves of Figure 2 were obtained choosing values for $\overline{\theta}$ from 4 to 20 and then using bisection to find the maximum value of ν for which the system is stable for each one of the four LMI conditions used here. For sake of performance comparison, observe that for tests based on affinely parameter-dependent Lyapunov functions, the conditions of Lemma 2 provide (with lower numerical complexity) better results than those from the conditions of [7]. Concerning tests based on quadratically parameter-dependent Lyapunov functions, the conditions of Theorem 1 provide (with lower numerical complexity) better estimates than those from the conditions of [13]. In an overall analysis, the conditions of Theorem 1 provide the less conservative results.

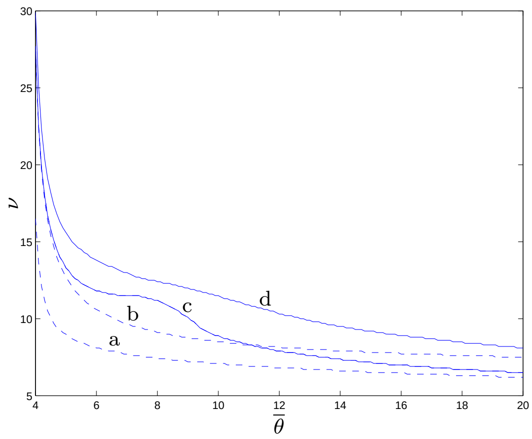

Figure 2: Bounds on the stability domain of system (33) obtained from the conditions given in [7] (curve a), from the conditions given in [13] (curve b), from the conditions of Lemma 2 (curve c) and using the conditions of Theorem 1 (curve d).

6 Conclusion

This paper provides sufficient LMI conditions relying on quadratically parameter-dependent Lyapunov functions for the analysis of the global asymptotic stability of the origin of linear systems affected by one uncertain time-varying parameter supposed to be bounded and with bounded time-derivatives. When the bound on the rates of variation of the parameter is given *a priori*, the existence of solution for an LMI test assures the system stability. Otherwise, a unidimensional search method associated with the feasibility of LMIs allows to estimate the maximum value of the bound on the time-derivatives of the parameter for which the stability is guaranteed. Numerical evaluation shows that the proposed conditions provide less conservative results than other LMI conditions in the literature which rely on affinely and on quadratically parameter-dependent Lyapunov functions. Although only systems affected by a single uncertain parameter are treated here, extensions to deal with systems affected by several uncertain parameters can be obtained following the methodology proposed in the paper.

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References

[1] H. L. S. Almeida, A. Bhaya, D. M. Falcão, and E. Kaszkurewicz. A team algorithm for robust stability analysis and control design of uncertain time-varying linear systems using piecewise quadratic Lyapunov functions. *International Journal of Robust and Nonlinear Control*, 11:357–371, 2001.

[2] P. Apkarian and H. D. Tuan. Parametrized LMIs in control theory. *SIAM Journal on Control and Optimization*, 38(4):1241–1264, May 2000.

[3] B. R. Barmish. Necessary and sufficient conditions for quadratic stabilizability of an uncertain system. *Journal of Optimization Theory and Applications*, 46(4):399–408, August 1985.

[4] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*. SIAM Studies in Applied Mathematics, Philadelphia, PA, 1994.

[5] G. Chesi, A. Garulli, A. Tesi, and A. Vicino. Parameter-depended homogeneous Lyapunov functions for robust stability of linear time-varying systems. In *Proceedings of the 43rd IEEE Conference on Decision and Control*, pages 4095–4100, Atlantis, Paradise Island, Bahamas, December 2004.

[6] G. Chesi, A. Garulli, A. Tesi, and A. Vicino. Robust analysis of LFR systems through homogeneous polynomial Lyapunov functions. *IEEE Transactions on Automatic Control*, 49(7):1211–1216, July 2004.

[7] P. Gahinet, P. Apkarian, and M. Chilali. Affine parameter-dependent Lyapunov functions and real parametric uncertainty. *IEEE Transactions on Automatic Control*, 41(3):436–442, March 1996.

[8] J. C. Geromel and P. Colaneri. Robust stability of time varying polytopic systems. *Systems & Control Letters*. To appear.

[9] T. Iwasaki. Robust performance analysis for systems with structured uncertainty. *International Journal of Robust and Nonlinear Control*, 6:85–99, March 1996.

[10] M. Johansson. *Piecewise Linear Control Systems — A Computational Approach*, volume 284 of *Lecture Notes in Control and Information Science*. Springer-Verlag, Heidelberg, Germany, 2003.

[11] H. J. Khalil. *Nonlinear Systems*. Macmillan Publishing Company, Singapore, 1992.

[12] V. F. Montagner and P. L. D. Peres. Robust stability and $\mathcal{H}_\infty$ performance of linear time-varying systems in polytopic domains. *International Journal of Control*, 77(15):1343–1352, October 2004.

[13] A. Trofino and C. E. de Souza. Biquadratic stability of uncertain linear systems. *IEEE Transactions on Automatic Control*, 46(8):1303–1307, August 2001.

[14] L. Xie, S. Shishkin, and M. Fu. Piecewise Lyapunov functions for robust stability of linear time-varying systems. *Systems & Control Letters*, 31(3):165–171, August 1997.