Calculus of variations on time scales with contingent epiderivatives

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ABSTRACT

We propose the use of the contingent epiderivative as a tool of differentiation on the calculus of variations on time scales. By introducing the problem

$$\mathcal{L}(y) = \int_{a}^{b} L(t, y^{*}(t), D_{\uparrow}y(t)(u))d_{\uparrow}t \rightarrow \min \quad y(a) = y_{a}, \quad y(b) = y_{b},$$

we are able to unify the two approaches explored so far in the literature of time scales. Indeed, for $u = 1$ problem (1) is reduced to the delta problem of the calculus of variations studied in [2, 3],

$$\int_{a}^{b} L(t, y^{\sigma}(t), y^{\Delta}(t))\Delta t \rightarrow \min , \quad y(a) = y_{a}, \quad y(b) = y_{b},$$

while for $u = -1$ we get the nabla variational problem on time scales [1, 5]:

$$\int_{a}^{b} L(t, y^{\rho}(t), -y^{\nabla}(t))\nabla t \rightarrow \min , \quad y(a) = y_{a}, \quad y(b) = y_{b}.$$

Moreover, studying (1) with $u = \pm 1$ as a two-objective optimization problem we are able to extend the ideas of [4] and prove new necessary and sufficient Pareto optimality conditions on time scales. Some illustrative examples are given.

Key words: contingent epiderivative, calculus of variations, time scales, Pareto optimality.

References


∗Supported by the post-doc fellowship SFRH/BPD/48439/2008 of the Portuguese Foundation for Science and Technology (FCT). On leave of absence from Faculty of Computer Science, Białystok Technical University, 15-351 Białystok, Poland, egirejko@pb.bialystok.pl.

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