Analyzing properties of interval-valued fuzzy coimplications obtained via the canonical interval representation

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ABSTRACT

The concept of duality, stating that something can or must coexist with its opposite, makes fuzzy logic seem natural, even inevitable. In fact, there are infinite degrees of uncertainty between the certainty of being and not being. This imperfection inherent to information represented in natural language has been treated mathematically using the theory of fuzzy sets. Thus, fuzzy logic introduces the ability to infer conclusions based on vague, ambiguous and qualitatively incomplete and inaccurate information [13]. In this context, the fuzzy-based systems have a way of reasoning similar to humans, representing the expressions of natural language in a very simple and intuitive manner, leading to the construction of expert systems easy to understand and maintain.

An additional important research area based on mathematical models for the uncertainty treatment considers the interval mathematics, providing methods which deliver results with guarantees [9]. Techniques of interval mathematics have been applied in the numerical computations related to approximations and rounding errors in computational procedures, providing the accuracy and reliability of computed results [15]. The use of interval arithmetic has achieved significant results in scientific computation, providing self-validating numerical algorithms with automatic error control in the floating point system, bounding the solution set [5].

Several extensions of fuzzy logic have been proposed in order to consider the imprecision modeled by the membership degree. Among them, this work focuses on the interval valued fuzzy set (IVFS), i.e. a theory where the membership degrees on a set are intervals in \( U = \{ [a, b] / 0 \leq a \leq b \leq 1 \} \) instead of single values in \( U = \{ x \in \mathbb{R} / 0 \leq x \leq 1 \} \). As an extension of traditional fuzzy set theory, the degrees of membership valued at \( U \) are approximations (for information partially unknown) of exact degrees valued at \( U \). In this case, not only the imprecision (lack of well-defined boundaries between clusters modeled), but also a feature of uncertainty (lack of information of these sets) can be analyzed. Since IVFS integrate fuzzy set theory with interval mathematics, it can be considered as base for the development of expert systems with emphasis on mathematical modeling in the treatment of data uncertainty, considering both the automatic error control and inaccuracy in numerical calculations [6].

The interval extension of fuzzy connectives studied in this work is based on the Canonical Interval Representation (CIR) introduced in [2, 3], restricted to the set \( U \). This interval representation of real functions always returns the smallest interval containing the image of the function, formalizing the notion of optimality. This work is focused on the study of interval-valued fuzzy coimpllications, characterized as the dual structure of interval-valued fuzzy im-
The analysis of properties and correlations between implications and coimplications has been studied, considering the important role they play in inference systems based on fuzzy logic and intuitionistic logic [16, 8, 7].

Similar to previous works in developing interval-valued coimplications [12], this work shows that the duality relation is preserved by the CIR operator stating the commutativity between classes of coimplications and interval-valued coimplications, including their corresponding dual constructions [11, 4]. This work also investigates extra properties that can be naturally extended to interval-valued fuzzy coimplications. Notice that some properties have two natural versions based on two natural partial orders on \(U\) and corresponding two continuity notions. The canonical representation of fuzzy coimplications is an inclusion-monotonic in both arguments. Thus, the aim of this work is to point out that from any fuzzy coimplication it is always possible to obtain canonically an interval fuzzy coimplication, in such a way that it is optimal and preserves the extra properties satisfied by the fuzzy coimplication.

**Keywords**: Interval fuzzy logic, Interval-valued fuzzy coimplication

**Referências**


