Internal wave models

Ailín Ruiz de Zárate  
Saulo Pomponet Oliveira
UFPR - Departamento de Matemática
Centro Politécnico, 81531-990, Caixa Postal 019081, Curitiba, PR
E-mail: ailin@ufpr.br, saulopo@ufpr.br,

André Nachbin
IMPA, Instituto de Matemática Pura e Aplicada
Estrada Dona Castorina 110, 22460-320, Rio de Janeiro, RJ
E-mail: nachbin@impa.br

Daniel Gregorio Alfaro Vigo
UFRJ, Instituto de Matemática, Departamento de Ciência da Computação
Rua Athos da Silveira Ramos, Bloco E, 21941-590, Caixa Postal 68530, Rio de Janeiro, RJ
E-mail: dgalfaro@dcc.ufrj.br

ABSTRACT

In the context of atmosphere and ocean dynamics, reduced strongly nonlinear one-dimen
dional models for the evolution of internal waves over topography were derived in [12, 13, 14]. These
reduced models aim at obtaining efficient numerical methods for a two-dimensional problem
with two layers containing inviscid, immiscible, incompressible and irrotational fluids of different
densities. The upper layer is shallow compared with the characteristic wavelength at the interface
of the two-fluid system, while the bottom region’s depth is comparable to the characteristic
wavelength. The non-linear evolution equations describe the behaviour of $\eta$ (the internal wave
elevation at the interface) and $u$ (the mean upper-velocity). These models are generalizations of
the one proposed in [4] for this water configuration. Some important aspects will be discussed
such as dispersion relations, the nonlocal effect of the Hilbert Transform on the strip and the use
of a conformal mapping to handle general topography profiles which introduces terrain following
coordinates and a variable coefficient accompanying each spatial derivative. This is an ongoing
research, we intend to use these models to study the interaction of large amplitude internal
waves with multiscale topography profiles. The dynamics include wave scattering, dispersion and
attenuation among other phenomena. The refocusing and stabilization of solitary waves for the
large levels of nonlinearity allowed by this kind of models are the goal of current research. These
models are relevant in oil recovery in deep ocean waters, where salt concentration and differences
in temperature generate stratification in such a way that internal waves can affect offshore
operations and submerged structures. On the other hand, the form drag of the topography
(from mountain ranges to smaller topographic features) is manifested in stratified effects such
as internal gravity waves which are important in the study of pollution dispersion in an urban
area, among other problems in atmosphere dynamics and weather prediction.

Keywords: Internal waves, Dispersive Strongly Nonlinear Models, Inhomogeneous media, Asymp-
totic theory

References


